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Nuclear Matrix Elements of $0\nu\beta\beta$-Decay: Possible Test of the Calculations

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Abstract

A possible model independent test of the theoretically calculated nuclear matrix elements of $0\nu\beta\beta$-decay is proposed. The test can be accomplished if $0\nu\beta\beta$-decay of three (or more) nuclei is observed. The selection of the nuclei for the next generation of $0\nu\beta\beta$-decay experiments should be done taking into account considerations regarding the possibility to test the nuclear matrix element calculations. The test proposed allows also to check the dominance of the Majorana mass mechanism of violation of the total lepton charge.

1 Introduction

The status of the problem of neutrino mixing changed drastically during the last several years: in the Super-Kamiokande (SK) atmospheric neutrino [1], SNO solar neutrino [2, 3] and KamLAND reactor antineutrino [4] experiments *model independent evidences of neutrino oscillations* were obtained. All neutrino oscillation data, except, the data of the LSND experiment [5] [2], can be described if we assume the existence of three-neutrino mixing in vacuum:

$$\nu_{lL}(x) = \sum_{i=1}^{3} U_{li}\,\nu_{iL}(x). \qquad (1)$$

Here $\nu_i(x)$ is the field on neutrino with mass m_i and U is the unitary PMNS [7, 8] mixing matrix.

The SK atmospheric neutrino data are best described in terms of two-neutrino $\nu_\mu \to \nu_\tau$ oscillations. From the analysis of the data the following best-fit values of the oscillation parameters were found [1]:

$$|\Delta m^2_{32}| = 2 \cdot 10^{-3}\mathrm{eV}^2, \quad \sin^2 2\theta_{23} = 1.0 \quad (\chi^2_{\min} = 170.8/170\,\mathrm{d.o.f.}). \qquad (2)$$

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[2] In the accelerator LSND experiment indications in favor of the transitions $\bar\nu_\mu \to \bar\nu_e$ with $(\Delta m^2)_{\mathrm{LSND}} \simeq 1\mathrm{eV}^2$ were obtained. The LSND results are being tested in the MiniBooNE experiment at Fermilab [6].

At the 90% C.L. one has:

$$1.3 \cdot 10^{-3} \leq |\Delta m^2_{32}| \leq 3.0 \cdot 10^{-3} \mathrm{eV}^2, \qquad \sin^2 2\theta_{23} > 0.9. \tag{3}$$

The results of all solar neutrino experiments can be explained by $\nu_e \to \nu_{\mu,\tau}$ transitions in matter. In the KamLAND experiment, $\bar{\nu}_e$ disappearance due to transitions $\bar{\nu}_e \to \bar{\nu}_{\mu,\tau}$ in vacuum was observed. From a global two-neutrino oscillation analysis of the solar and KamLAND data (performed under the assumption of CPT-invariance), the following best-fit values of the relevant oscillation parameters were obtained [3]:

$$\Delta m^2_{21} = 7.1 \cdot 10^{-5} \mathrm{eV}^2, \qquad \tan^2 \theta_{12} = 0.41. \tag{4}$$

In a similar 3-neutrino oscillation analysis of the solar neutrino, KamLAND and CHOOZ [9] data, performed in [10], it was found that at 90% C.L. one has:

$$\begin{array}{c} 5.6 \cdot 10^{-5} \leq \Delta m^2_{21} \leq 9.2 \cdot 10^{-5} \mathrm{eV}^2, \quad 0.23 \leq \sin^2 \theta_{12} \leq 0.38, \quad \text{for} \quad \sin^2 \theta_{13} = 0.0, \\ 6.1 \cdot 10^{-5} \leq \Delta m^2_{21} \leq 8.5 \cdot 10^{-5} \mathrm{eV}^2, \quad 0.25 \leq \sin^2 \theta_{12} \leq 0.36, \quad \text{for} \quad \sin^2 \theta_{13} = 0.04, \end{array} \tag{5}$$

where θ_{13} is the mixing angle limited by the reactor CHOOZ and Palo Verde experiments [9, 11]. The negative results of the CHOOZ [9] and Palo Verde [11] experiments are very important for understanding the pattern of neutrino mixing and oscillations. In these experiments no disappearance of $\bar{\nu}_e$ was observed. From the 90% C.L. exclusion curve obtained from the analysis of the data of the CHOOZ experiment, the following bound can be derived

$$\sin^2 \theta_{13} < 5 \cdot 10^{-2}. \tag{6}$$

The same result was obtained in [10] in a global 3-neutrino oscillation analysis of the solar, KamLAND and CHOOZ data with $|\Delta m^2_{32}|$ taken to lie in the interval eq. (3).

There are two general theoretical possibilities for the fields of neutrinos with definite masses $\nu_i(x)$ (see, e.g., [12]):

1. If the total lepton charge $L = L_e + L_\mu + L_\tau$ is conserved, $\nu_i(x)$ are *Dirac fields* of neutrinos ν_i (L=1) and antineutrinos $\tilde{\nu}_i$ (L=-1).

2. If there are no conserved lepton charges, $\nu_i(x)$ are *Majorana fields* which satisfy the condition

$$\nu_i^c(x) = C\, \bar{\nu}_i^T(x) = \nu_i(x), \tag{7}$$

   C being the charge conjugation matrix, and neutrinos ν_i are *Majorana particles.*

The solution of the problem of the nature of the neutrinos with definite mass - Dirac or Majorana, will be of fundamental importance for the understanding of the origin of small neutrino masses and of the pattern of neutrino mixing.

The investigation of neutrino oscillations does not permit to determine the nature of massive neutrinos [13, 14]. Processes in which the total lepton charge L is not conserved and changes by two units, must be studied for that purpose. These processes are allowed if the massive neutrinos are Majorana particles. Experiments searching for neutrinoless double β- decay of even-even nuclei (see, e.g., [15, 16, 17, 18, 19, 20, 21]),

$$(\mathrm{A}, \mathrm{Z}) \rightarrow (\mathrm{A}, \mathrm{Z}+2) + \mathrm{e}^- + \mathrm{e}^- . \qquad (8)$$

have the highest sensitivity to the nonconservation of the total lepton charge and to Majorana neutrino masses. The matrix element of $0\nu\beta\beta$ -decay is proportional to the effective Majorana mass (see, e.g., [12, 22]):

$$|m_{ee}| = \left| m_1 |U_{\mathrm{e}1}|^2 + m_2 |U_{\mathrm{e}2}|^2 \; e^{i\alpha_{21}} + m_3 |U_{\mathrm{e}3}|^2 \; e^{i\alpha_{31}} \right| \; , \qquad (9)$$

where U_{ej}, $j = 1, 2, 3$, are the elements of the first row of the PMNS neutrino mixing matrix U, $m_j > 0$ is the mass of the Majorana neutrino ν_j, and α_{21} and α_{31} are two Majorana CP-violating phases [13, 23]. One can express [24] (see also, e.g., [25, 22]) the two heavier neutrino masses and the elements $|U_{ej}|$ in $|m_{ee}|$ in terms of the lightest neutrino mass, Δm^2_{21}, Δm^2_{32}, and of θ_{12} and θ_{13}, respectively.

The results of a large number of experiments searching for $0\nu\beta\beta$ -decay are available at present (see, e.g., [20, 26, 27, 28]). In the Heidelberg-Moscow experiment [29] the most stringent lower bound on the half-life of $0\,\nu\beta\,\beta$-decay of ^{76}Ge has been obtained [3]:

$$T^{0\nu}_{1/2} > 1.9 \cdot 10^{25}\,\mathrm{y} \quad (90\%\,\mathrm{C.L.}). \qquad (10)$$

In the cryogenic experiment CUORICINO the following lower bound on the $0\,\nu\beta\,\beta$-decay half-life of ^{130}Te was recently reported [33]:

$$T^{0\nu}_{1/2} > 7.5 \cdot 10^{23}\,\mathrm{y} \qquad (11)$$

Taking into account the result of different calculations of the relevant nuclear matrix elements, the following upper bounds on the effective Majorana mass $|m_{ee}|$ can be inferred from the limits (10) and (11):

$$|m_{ee}| < (0.3 - 1.2) \;\; \mathrm{eV}, \qquad |\mathrm{m_{ee}}| < (0.3 - 1.7) \;\; \mathrm{eV}. \qquad (12)$$

The NEMO3 experiment searching for $0\,\nu\beta\,\beta$-decay of a number of different nuclei (^{100}Mo, ^{82}Se, etc.) and aiming at a precision of $|m_{ee}| \sim 0.1$ eV, is successfully taking data at present [34].

Many new projects of experiments searching for neutrinoless double β-decay of ^{76} Ge, ^{136} Xe, ^{130} Te, ^{100} Mo and other nuclei, are under research and development at present (see, e.g., [19, 20, 27, 35]). The goal of the future experiments is to reach a sensitivity

$$|m_{ee}| \simeq \;\; \text{few} \times 10^{-2} \;\mathrm{eV}. \qquad (13)$$

In order to obtain information about the effective Majorana mass $|m_{ee}|$ from the results of the $0\nu\beta\beta$-decay experiments, the corresponding $0\nu\beta\beta$-decay nuclear matrix elements must be known. A large number of calculations of the nuclear matrix elements of the $0\nu\beta\beta$-decay exist in the literature (see, e.g., [16, 17, 19]). The results of different calculations differ by a factor three or more. We will propose here a possible test of the nuclear matrix elements calculations. It can be accomplished if $0\nu\beta\beta$-decay of *several nuclei* will be observed in the future experiments.

[3] Indications for $0\,\nu\beta\,\beta$-decay of ^{76}Ge with a rate corresponding to 0.11 eV $\leq |m_{ee}| \leq$ 0.56 eV (95% C.L.), are claimed to have been obtained in [30]. The results announced in [30] have been criticized in [31]. Even stronger evidence has been reported recently in [32]. These claims will be checked in the currently running and future $0\,\nu\beta\,\beta$-decay experiments. However, it may take a very long time before comprehensive checks could be completed.

2 The effective Majorana mass

The neutrino oscillation data allow to predict the possible ranges of values of the effective Majorana mass. The prediction depend strongly on the character of the neutrino mass spectrum and on the value of the lightest neutrino mass (see, e.g., [22, 25, 36, 37, 38]). We will summarize here briefly the main results for the three possible types of neutrino mass spectrum [39, 40].

1. *Normal hierarchical neutrino mass spectrum*:

$$m_1 \ll m_2 \ll m_3 \tag{14}$$

For the effective Majorana mass we have in this case the following upper bound

$$|m_{ee}| \lesssim \left(\sin^2\theta_{12}\sqrt{\Delta m^2_{21}} + \sin^2\theta_{13}\sqrt{\Delta m^2_{32}}\right). \tag{15}$$

Using the 90% C.L. ranges (3) and (5) of the oscillation parameters and the CHOOZ bound (6), for the effective Majorana mass one finds [39]

$$|m_{ee}| \lesssim 5.5 \cdot 10^{-3}\ \text{eV}, \tag{16}$$

This bound is significantly smaller than the expected sensitivity of the future $0\nu\beta\beta$-decay experiments. The observation of the $0\nu\beta\beta$-decay in the next generation of experiments might exclude normal hierarchical neutrino mass spectrum.

2. *Inverted hierarchical neutrino mass spectrum*:

$$m_3 \ll m_1 < m_2. \tag{17}$$

For the effective Majorana mass we have in this case [36, 25]:

$$|m_{ee}| \simeq \sqrt{|\Delta m^2_{32}|}\ (1 - \sin^2 2\,\theta_{12}\,\sin^2\alpha)^{\frac{1}{2}}, \tag{18}$$

where $\alpha = \alpha_{21}$ is the Majorana $CP-$ violating phase. From eq. (18) we obtain the range

$$\sqrt{|\Delta m^2_{32}|}\ \cos 2\,\theta_{12} \lesssim |m_{ee}| \lesssim \sqrt{|\Delta m^2_{32}|}, \tag{19}$$

where the upper and lower bounds corresponds to the case of CP conservation and the same and opposite CP-parities of neutrinos ν_1 and ν_2. Using the 90% C.L. allowed values of the parameters, eqs. (3) and (5), for the effective Majorana mass one finds [39]:

$$10^{-2}\ \text{eV} \lesssim |\mathrm{m_{ee}}| \lesssim 5.5 \cdot 10^{-2}\ \text{eV} \tag{20}$$

Thus, if the neutrino mass spectrum is of the inverted hierarchical type and the massive neutrino are Majorana particles, $0\nu\beta\beta$-decay can be observed in the experiments of next generation.

3. *Quasi-degenerate neutrino mass spectrum*:

$$m_1 \simeq m_2 \simeq m_3, \quad m^2_{1,2,3} \gg |\Delta m^2_{32}| \ . \tag{21}$$

The effective Majorana mass is given in this case by eq. (18) in which $\sqrt{|\Delta m^2_{32}|}$ is replaced by $m_{\min}$, where $m_{\min}$ is the lightest neutrino mass. For the effective Majorana mass we have the range [39]

$$0.22\, m_{\min} \lesssim |m_{ee}| \lesssim m_{\min}. \tag{22}$$

In the case of quasi-degenerate neutrino mass spectrum, the effective Majorana mass depends essentially on two parameters: the lightest neutrino mass $m_{\min}$ and the CP-violating parameter $\sin^2\alpha$. From the measurement of $|m_{ee}|$, the following range for the lightest neutrino mass can be obtained:

$$|m_{ee}| \lesssim m_{\min} \lesssim 4.6 \ \ |m_{ee}| \ . \tag{23}$$

If the lightest neutrino mass $m_{\min}$ will be determined in the tritium $\beta-$decay experiment KATRIN [41] which is under preparation at present, or from cosmological and astrophysical observations (see, e.g., [42]), the data of $0\nu\beta\beta$ experiments can be used to get information about the Majorana CP-violating phases [36, 22, 43, 44].

Neutrinoless double β-decay is a unique process. The observation of this process would be a proof that the total lepton charge is not conserved and massive neutrinos ν_i are Majorana particles. As we have seen in this Section, the precise measurement of the parameter $|m_{ee}|$ would allow to draw important conclusions about the character of neutrino mass spectrum, the lightest neutrino mass and the CP-violation associated with the Majorana neutrinos (for a more detailed discussion see, e.g., [45]). However, from the data of the $0\nu\beta\beta$-decay experiments only the product of $|m_{ee}|$ and the corresponding nuclear matrix element can be determined. In the next section we will briefly discuss the problem of the calculation of the nuclear matrix elements of $0\nu\beta\beta-$decay.

3 The Problem of Calculation of Nuclear Matrix Elements

If neutrinoless double β-decay is due to the Majorana neutrino mixing (1) *only*, it proceeds via exchange of a virtual neutrino and is a process of second order in the Fermi constant G_F. The nuclear matrix element (NME) of the $0\nu\beta\beta$-decay of a given even-even nuclei cannot be related to other observables and has to be calculated. The calculation of NME is a complicated problem. One of the problems of the calculations is connected with a large number of states of the intermediate odd-odd nuclei, which are important due to relatively large average momentum of the virtual neutrino.

Many calculations of NME exist in literature (see, e.g., the review articles [16, 17, 19, 46]). Two basic methods of calculations of NME are used at present: nuclear shell model (NSM) and quasiparticle random phase approximation (QRPA).

The nuclear shell model is attractive from physical point of view: there are many spectroscopic data in favor of shell structure of nuclei (spins and parities of nuclei, binding energies of magic nuclei, etc.) [47]. However, only rather limited set of one-particles states of valent nucleons can

Table 1: Half-life of the $0\nu\beta\beta$-decay for $|m_{ee}| = 5 \cdot 10^{-2}$ eV. The nuclear matrix elements were taken from the compilation in ref. [46].

Nucleus	$T^{0\nu}_{1/2}$years
^{76}Ge	$1.4 \cdot 10^{27} - 1.5 \cdot 10^{29}$
^{100}Mo	$1.7 \cdot 10^{26} - 5.9 \cdot 10^{30}$
^{130}Te	$7.7 \cdot 10^{26} - 3.4 \cdot 10^{27}$
^{136}Xe	$2.7 \cdot 10^{27} - 1.7 \cdot 10^{28}$

be taken into account because of practical computational reasons. It is difficult to estimate the accuracy of the shell model calculations.

The most popular method of calculation of the NME of $0\nu\beta\beta$ -decay is QRPA [16, 17]. This method allows to use as a basis a large number of one-particle states and to take into account all intermediate states. Important parameters of QRPA are the constant of particle-hole interaction, g_{ph}, and the constant of particle-particle interaction, g_{pp}. The constant g_{ph} can be fixed from a fit of the energy of the giant Gamov-Teller resonance. The constant g_{pp} is a free parameter.

There are many models based on the QRPA approach (see, reviews [16, 17, 19, 46]). The results of the calculations of NME of $0\nu\beta\beta$ -decay performed by different authors differ quite significantly. The variety of results of the calculations is illustrated by Table 1 in which ranges of the values of the half-life of the $0\nu\beta\beta$-decay of different nuclei are presented for $|m_{ee}| = 5 \cdot 10^{-2}$ eV.

Recently, in the framework of QRPA, a new procedure of calculation of the NME was proposed [48]. For a fixed value of the constant g_{ph} and the values of the parameter g_{pp}, determined from the measured $2\nu\beta\beta$-decay half-life, the $0\nu\beta\beta$-decay nuclear matrix elements of several nuclei were calculated. In [48] results were derived for three different sets of the one-particle states and for three different nucleon-nucleon potentials, and it was shown that the NME of $0\nu\beta\beta$-decay depends weakly on the number of one-particle states and on the nucleon-nucleon potential used in the calculations (the NME for each of the nuclei thus calculated varies by not more than 10%).

Another approach was proposed in [46]. In this paper all parameters of the QRPA model, including the g_{pp} constant, were fixed from data on β-decay of nuclei which are close to the even-even nuclei of interest for the $0\nu\beta\beta$ -decay study.

In spite of the recent progress in the calculation of NME, it is not possible at present to estimate the real accuracy of the calculations. It is important to find a possibility to check the calculations of NME by a direct comparison with experimental data [49]. Such a possibility will be discussed in the next Section.

4 Possible Test of the NME Calculations

If the Majorana neutrino mixing (1) is the mechanism of $0\nu\beta\beta$-decay, the matrix element of the process has the following general form (see, e.g., [15, 12])

$$\langle f|(S-1)|i\rangle = N\, m_{ee}\, M^{0\,\nu}(A,Z)\,\delta(E_f - E_i). \tag{24}$$

Here N is a product of known factors and $M^{0\,\nu}(A,Z)$ is the nuclear matrix element of interest. The neutrino masses enter into the matrix element $\langle f|(S-1)|i\rangle$ through the effective Majorana mass m_{ee} given by (9) and the neutrino propagator $(q^2-m_i^2)^{-1}$ which is included in $M^{0\,\nu}(A,Z)$, q being the momentum of the virtual neutrino. For small neutrino masses (smaller than the binding energy of nucleons in nuclei ~ 10 MeV), the neutrino masses in the propagator can be safely neglected. The matrix element $M^{0\,\nu}(A,Z)$ depends in this case only on the nuclear properties and strong interaction.

The half-life of the $0\nu\beta\beta$ is given by the expression [4]:

$$\frac{1}{T^{0\,\nu}_{1/2}(A,Z)} = |m_{ee}|^2\,|M^{0\,\nu}(A,Z)|^2\,G^{0\,\nu}(E_0,Z), \tag{25}$$

where $G^{0\,\nu}(E_0,Z)$ is known phase-space factor (E_0 is the energy release). If we use a model M of the calculation of NME we have

$$|m_{ee}|^2_M(A,Z) = \frac{1}{T^{0\,\nu}_{1/2}(A,Z)\,|M^{0\,\nu}_M(A,Z)|^2\,G^{0\,\nu}(E_0,Z)}. \tag{26}$$

Let us assume that neutrinoless double β-decay of *several* nuclei is observed. The effective Majorana mass $|m_{ee}|$ cannot depend on the parent nucleus. Thus, if the light Majorana neutrino exchange is the dominant mechanism of $0\nu\beta\beta$-decay, the model M of the calculation of the nuclear matrix elements can be correct only if the relations

$$|m_{ee}|^2_M(A_1,Z_1) \simeq |m_{ee}|^2_M(A_2,Z_2) = ... \tag{27}$$

hold, where $|m_{ee}|^2_M(A_j,Z_j)$ is the value of $|m_{ee}|^2$ obtained from the $0\nu\beta\beta$-decay half-life of the nucleus (A_j,Z_j) using the model M.

Consider different models of calculation of NME. From eq. (25) it follows that for a given parent nucleus the product $|m_{ee}|^2_M(A,Z)|\,|M^{0\,\nu}_M(A,Z)|^2$ does not depend on the model. Thus, for different models and the same nucleus we have

$$|m_{ee}|^2_{M_1}(A,Z)|\,|M^{0\,\nu}_{M_1}(A,Z)|^2 = |m_{ee}|^2_{M_2}(A,Z)|\,|M^{0\,\nu}_{M_2}(A,Z)|^2 = ... \tag{28}$$

For two different models we have the relation

$$|m_{ee}|^2_{M_2}(A,Z) = \eta^{M_2;M_1}(A,Z)\,|m_{ee}|^2_{M_2}(A,Z) \tag{29}$$

where

$$\eta^{M_2;M_1}(A,Z) = \frac{|M^{0\,\nu}_{M_1}(A,Z)|^2}{|M^{0\,\nu}_{M_2}(A,Z)|^2}\,. \tag{30}$$

[4] It follows from this expression that the relative accuracy of determination of the parameter $|m_{ee}|$ (for any value of NME) is two times better that the relative accuracy of the measurement of the half-life of $0\nu\beta\beta$-decay:

$$\frac{\Delta|m_{ee}|}{|m_{ee}|} = \frac{1}{2}\,\frac{\Delta T^{0\,\nu}_{1/2}}{T^{0\,\nu}_{1/2}}$$

.

Table 2: The parameter $\eta^{M_i;M_k}(A,Z)$, determined by eq. (30), for the nuclear matrix elements of $0\nu\beta\beta$- decay, calculated in ref. [50] (M_1), in ref. [48] (M_2) and in ref. [46] (M_3).

Nucleus	$\eta^{M_2;M_1}$	$\eta^{M_3;M_1}$	$\eta^{M_2;M_3}$
^{76}Ge	0.37	0.19	1.93
^{82}Se	—	0.38	—
^{100}Mo	—	—	6.56
^{130}Te	0.74	0.10	7.32
^{136}Xe	0.53	0.02	22.42

In Table 2 we present the values of the coefficient $\eta(A,Z)$ for the case of the matrix elements calculated in [50] (NSM), in [48] (QRPA), and in [46] (QRPA, different model).

We see from Table 2 that the coefficient $\eta(A,Z)$ depends rather strongly on (A,Z). This means that if for one model the relation (27) is satisfied, other models, in principle, can be excluded. However, the observation of $0\nu\beta\beta$-decay of only two nuclei might not allow to distinguish between different models. For example, the observation of the $0\nu\beta\beta$-decay of ^{100}Mo and ^{130}Te will not allow to distinguish the QRPA models of ref. [48] and of ref. [46] (the difference between the values of the coefficient $\eta(A,Z)$ for these two nuclei is about 10%). The values of the effective Majorana mass which can be obtained from the observation of the $0\nu\beta\beta$-decay of these two nuclei, if one uses the models of ref. [48] and of ref. [46], will differ by a factor of ~ 2.6. The observation of neutrinoless double β-decay of at least three nuclei would be an important tool in the solution of the problem of NME. Table 2 suggests that the observation of the $0\nu\beta\beta$- decay of ^{76}Ge, ^{130}Te and ^{136}Xe would solve the problem..

If relations (27) are satisfied for some model M, this would mean that the corresponding value of the effective Majorana mass $|m_{ee}|_M$ can be different from the true value $|m_{ee}|_0$ by a constant factor:

$$|m_{ee}|_M = \beta\,|m_{ee}|_0. \tag{31}$$

From this relation it follows that

$$|M^{0\nu}_M(A,Z)|_M = \frac{1}{\beta}\,|M^{0\nu}_M(A,Z)|_0, \tag{32}$$

where $|M^{0\nu}_M(A,Z)|_0$ is the true value of the NME (which, of course, we do not know). For the test we are proposing it is important that nuclear matrix elements depend rather strongly on (A,Z) .

It looks quite improbable that relation (32) with one and the same constant β would be valid for nuclei with different properties and different NME, and especially for three different nuclei. Thus, if relation (27) is satisfied for some model of calculations of nuclear matrix elements, the corresponding value of the effective Majorana mass would most likely be the true value.

One last remark. We have assumed that the mechanism of $0\nu\beta\beta$-decay is the Majorana neutrino mixing, eq. (1). There exist, however, many other mechanisms of nonconservation of the total lepton charge and $0\nu\beta\beta$-decay, like SUSY with violation of $R-$parity, etc. (see, e.g., the review articles [16, 17]). These additional mechanisms modify the neutron-proton-electron vertexes and include exchange not only of light neutrino but also of heavy particles (see, e.g., [51]). Because the nuclear matrix elements of different operators have different A, Z dependence, in the case of additional mechanisms the factorization property of $0\nu\beta\beta-$decay matrix elements, eq. (24), will not be valid and relation (27), in general, will not be satisfied. Thus, if relation (27) will be found to hold for a given model, this also will be a strong indication in favor of dominance of the Majorana neutrino mixing mechanism of lepton charge non-conservation.

5 Conclusions

After the discovery of neutrino masses and neutrino mixing the problem of *the nature of neutrinos with definite masses* ν_i became one of the fundamental problems in the studies of neutrino mixing. The determination of the nature of massive neutrinos ν_i will have a profound implications for the understanding of the mechanism of generation of neutrino masses and mixing. The measurement of the effective Majorana mass would allow to obtain an important and unique information about the character of the neutrino mass spectrum, the lightest neutrino mass and possibly on the Majorana CP-violating phases.

From the measured half-life of $0\nu\beta\beta$-decay only the product of the effective Majorana mass and the corresponding nuclear matrix element can be obtained. The results of the different existing calculations of the $0\nu\beta\beta$-decay nuclear matrix elements vary significantly (see Table 1). The improvement of the calculations of the nuclear matrix elements is a very important and challenging problem. We have discussed here a possible method which could allow to test the models of calculation of NME of the $0\nu\beta\beta$-decay via comparison of the results of calculations with experimental data. The method is based on the factorization property of the matrix element of $0\nu\beta\beta$-decay and requires the observation of the $0\nu\beta\beta$-decay of several nuclei.

The nuclear matrix elements of the $0\nu\beta\beta$-decay cannot be related to other observables. We do not see at present any alternative possibility to confront the results of the NME calculations with experimental data in a model independent way.

New experiments searching for $0\nu\beta\beta$-decay of ^{130}Te, ^{76}Ge, ^{136}Xe, ^{100}Mo and other nuclei are in preparation at present. From the point of view of the problem of NME, it is very desirable that these projects will be realized *for at least three different nuclei. The selection of the nuclei should be done taking into account also the considerations discussed above regarding the possibility to test the nuclear matrix element calculations.*

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