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The Absolute Neutrino Mass Scale, Neutrino Mass Spectrum, Majorana CP-Violation and Neutrinoless Double-Beta Decay

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Abstract

Assuming 3-ν mixing, massive Majorana neutrinos and neutrinoless double-beta ($(\beta\beta)_{0\nu}$-) decay generated only by the $(V-A)$ charged current weak interaction via the exchange of the three Majorana neutrinos, we briefly review the predictions for the effective Majorana mass $|<\!m\!>|$ in $(\beta\beta)_{0\nu}$-decay and reanalyse the physics potential of future $(\beta\beta)_{0\nu}$-decay experiments to provide information on the type of neutrino mass spectrum, the absolute scale of neutrino masses, and Majorana CP-violation in the lepton sector. Using as input the most recent experimental results on neutrino oscillation parameters and the prospective precision that can be achieved in future measurements of the latter, we perform a statistical analysis of a $(\beta\beta)_{0\nu}$-decay half-life measurement taking into account experimental and theoretical errors, as well as the uncertainty implied by the imprecise knowledge of the corresponding nuclear matrix element (NME). We show, in particular, how the possibility to discriminate between the different types of neutrino mass spectra and the constraints on the absolute neutrino mass scale depend on the mean value and the experimental error of $|<\!m\!>|$ and on the NME uncertainty. The constraints on Majorana CP-violation phases in the neutrino mixing matrix, which can be obtained from a measurement of $|<\!m\!>|$ and additional data on the sum of neutrino masses, are also investigated in detail. We estimate the required experimental accuracies on both types of measurements, and the required precision in the NME permitting to address the issue of Majorana CP-violation in the lepton sector.

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1 Introduction

There has been remarkable progress in the studies of neutrino oscillations in the last several years. The experiments with solar, atmospheric, reactor and accelerator neutrinos [1–7] have provided compelling evidence for the existence of neutrino oscillations caused by nonzero neutrino masses and neutrino mixing [1]. The latest addition to these results are the Super-Kamiokande (SK) data on the L/E-dependence of the (essentially multi-GeV) μ-like atmospheric neutrino events [9], L and E being the distance traveled by neutrinos and the neutrino energy, and the new spectrum data of the KamLAND (KL) and K2K experiments [10, 11]. For the first time the data directly exhibit the effects of the oscillatory dependence on L/E and E of the probabilities of ν-oscillations in vacuum [12]. As a result of these magnificent developments, the oscillations of solar ν_e, atmospheric ν_μ and $\bar{\nu}_\mu$, accelerator ν_μ (at $L \sim 250$ km) and reactor $\bar{\nu}_e$ (at $L \sim 180$ km), driven by non-zero ν-masses and ν-mixing, can be considered as practically established.

The evidences for ν-oscillations obtained in the solar and atmospheric neutrino and KL and K2K experiments imply the existence of 3-ν mixing in the weak charged-lepton current:

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}}, \quad l = e, \mu, \tau, \tag{1}$$

where ν_{lL} are the flavour neutrino fields, $\nu_{j\mathrm{L}}$ is the field of neutrino ν_j having a mass m_j and U is the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) mixing matrix [13], $U \equiv U_{\mathrm{PMNS}}$. All existing ν-oscillation data, except the data of the LSND experiment [8], can be described assuming 3-ν mixing in vacuum; we will consider this possibility in what follows [2].

The PMNS matrix can be parametrized by 3 angles, and, depending on whether the massive neutrinos ν_j are Dirac or Majorana particles, by 1 or 3 CP-violation (CPV) phases [17, 18]. In the standardly used parametrization (see, e.g. [19]), U_{PMNS} has the form:

$$U_{\mathrm{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13}e^{i\delta} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}e^{i\delta} \end{pmatrix} \mathrm{diag}(1, e^{i\frac{\alpha_{21}}{2}}, e^{i\frac{\alpha_{31}}{2}})\,, \tag{2}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, the angles $\theta_{ij} = [0, \pi/2]$, $\delta = [0, 2\pi]$ is the Dirac CPV phase and α_{21}, α_{31} are two Majorana CPV phases [17, 18]. One can identify the neutrino mass squared difference responsible for solar neutrino oscillations, $\Delta m^2_\odot$, with $\Delta m^2_{21} \equiv m^2_2 - m^2_1$, $\Delta m^2_\odot = \Delta m^2_{21} > 0$. The neutrino mass squared difference driving the dominant $\nu_\mu \to \nu_\tau$ ($\bar{\nu}_\mu \to \bar{\nu}_\tau$) oscillations of atmospheric ν_μ ($\bar{\nu}_\mu$) is then given by $|\Delta m^2_{\mathrm{A}}| = |\Delta m^2_{31}| \cong |\Delta m^2_{32}| \gg \Delta m^2_{21}$. The corresponding solar and atmospheric neutrino mixing angles, $\theta_\odot$ and θ_{A}, coincide with θ_{12} and θ_{23}, respectively. The angle θ_{13} is limited by the data from the CHOOZ and Palo Verde experiments [20].

[1] Indications for ν-oscillations were reported also by the LSND collaboration [8].

[2] The interpretation of LSND data in terms of $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations requires $(\Delta m^2)_{\mathrm{LSND}} \simeq 1$ eV^2. The minimal 4-neutrino mixing scheme, which could incorporate the LSND indications for $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations, is strongly disfavored by the data [14]. The ν-oscillation explanation of the LSND results is possible, assuming 5-neutrino mixing [15]. The LSND results are being tested in the MiniBooNE experiment [16].

Thus, the basic phenomenological parameters characterising the 3-ν mixing are: i) the 3 angles θ_{12}, θ_{23}, θ_{13}, ii) depending on the nature of massive neutrinos 1 Dirac (δ) or 1 Dirac + 2 Majorana ($\delta, \alpha_{21}, \alpha_{31}$) CPV phases, and iii) the 3 neutrino masses, m_1, m_2, m_3. Getting precise information about the ν-mixing parameters is of fundamental importance for understanding the origin of neutrino mixing (see, e.g. [21]).

The existing neutrino oscillation data allow us to determine $\Delta m^2_\odot$, $|\Delta m^2_{\rm A}|$, $\sin^2\theta_\odot$ and $\sin^2 2\theta_{\rm A}$ with a relatively good precision and to obtain rather stringent limits on $\sin^2\theta_{13}$ (see, e.g. [5, 10, 14, 22, 23]). The data imply that $\Delta m^2_\odot = \Delta m^2_{21} \sim 8.0 \times 10^{-5}$ eV^2, $|\Delta m^2_{\rm A}| \sim 2.2 \times 10^{-3}$ eV^2, $\sin^2\theta_\odot \sim 0.30$, $\sin^2 2\theta_{\rm A} \sim 1$ and $\sin^2\theta_{13} < 0.05$. The sign of $\Delta m^2_{\rm A}$, as is well known, cannot be determined from the present (SK atmospheric neutrino and K2K) data. In the case of 3-ν mixing the two possibilities, $\Delta m^2_{31(32)} > 0$ or $\Delta m^2_{31(32)} < 0$ correspond to two different types of ν-mass spectrum:
– *with normal hierarchy (or ordering)*, $m_1 < m_2 < m_3$, $\Delta m^2_{\rm A} = \Delta m^2_{31} > 0$, and
– *with inverted hierarchy (ordering)* [3], $m_3 < m_1 < m_2$, $\Delta m^2_{\rm A} = \Delta m^2_{32} < 0$.
Depending on the sign of $\Delta m^2_{\rm A}$, $\text{sgn}(\Delta m^2_{\rm A})$, and the value of the lightest neutrino mass, $\min(m_j)$, the ν-mass spectrum can be

- *Normal Hierarchical (NH)*:
  $m_1 \ll m_2 \ll m_3$, with $m_2 \cong \sqrt{\Delta m^2_\odot} \sim 0.009$ eV and $m_3 \cong \sqrt{|\Delta m^2_{\rm A}|} \sim 0.047$ eV;

- *Inverted Hierarchical (IH)*:
  $m_3 \ll m_1 < m_2$, with $m_{1,2} \cong \sqrt{|\Delta m^2_{\rm A}|} \sim 0.047$ eV;

- *Quasi-Degenerate (QD)*:
  $m_1 \cong m_2 \cong m_3$, with $m_1 \cong m_2 \cong m_3 \cong m_0$, $m_j^2 \gg |\Delta m^2_{\rm A}|$, $m_0 \gtrsim 0.10$ eV.

The precision on the mixing angles θ_{21}, θ_{23}, θ_{13}, and on Δm^2_{21} and $|\Delta m^2_{31}|$, can be significantly improved in future ν-oscillation experiments (see, e.g. [26–30]). The sign of Δm^2_{31} can be determined by studying oscillations of neutrinos and antineutrinos, say, $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$, in which matter effects are sufficiently large. This can be done in long-baseline ν-oscillation experiments running both with neutrino and antineutrino beams (see, e.g. [31]) or in the neutrino mode only [32, 33]. If $\sin^2 2\theta_{13} \gtrsim 0.05$ and $\sin^2\theta_{23} \gtrsim 0.50$, information on $\text{sgn}(\Delta m^2_{31})$ might be obtained in atmospheric neutrino experiments by investigating the effects of the subdominant transitions $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ and $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ of atmospheric neutrinos that traverse the Earth [34, 35].

The neutrino oscillation experiments, however, cannot provide information on the absolute scale of neutrino masses (or on $\min(m_j)$) and thus on the possible hierarchical structure (NH, IH, QD, etc.) of the neutrino mass spectrum. The oscillations of flavour neutrinos, $\nu_l \rightarrow \nu_{l'}$ and $\bar{\nu}_l \rightarrow \bar{\nu}_{l'}$, $l, l' = e, \mu, \tau$, are insensitive to the nature—Dirac or Majorana—of massive neutrinos ν_j; they are insensitive to the Majorana CPV phases $\alpha_{21,31}$ [17, 36]. If ν_j are Majorana fermions, getting experimental information about the Majorana CPV phases

[3] In the convention we use (called A), the neutrino masses are not ordered in magnitude according to their index number: $\Delta m^2_{31} < 0$ corresponds to $m_3 < m_1 < m_2$. We can also always number the neutrinos with definite mass, in such a way that [24] $m_1 < m_2 < m_3$. In this convention (called B) we have in the case of the inverted hierarchy spectrum: $\Delta m^2_\odot = \Delta m^2_{32}$, $\Delta m^2_{\rm A} = \Delta m^2_{31}$. Convention B is used, e.g. in [19, 25].

in $U_{\rm PMNS}$ would be a remarkably challenging problem [4] [24, 39–41].

Establishing whether ν_j are Dirac fermions possessing distinct antiparticles, or are Majorana fermions, i.e. spin 1/2 particles that are identical with their antiparticles, is of fundamental importance for understanding the underlying symmetries of particle interactions and the origin of ν-masses. Let us recall that neutrinos ν_j with definite mass will be Dirac fermions if particle interactions conserve some additive lepton number, e.g. the total lepton charge $L = L_e + L_\mu + L_\tau$. If no lepton charge is conserved, the neutrinos ν_j will be Majorana fermions (see, e.g. [42]). The observed patterns of ν-mixing and of $|\Delta m^2_{\rm A}|$ and $\Delta m^2_\odot$ can be related to Majorana ν_j and the existence of an *approximate* symmetry corresponding to the conservation of the lepton charge $L' = L_e - L_\mu - L_\tau$ [43, 44]. The massive neutrinos are predicted to be of Majorana nature by the see-saw mechanism of neutrino mass generation [45], which also provides an attractive explanation of the smallness of neutrino masses and, through the leptogenesis theory [38], of the observed baryon asymmetry of the Universe. Determining the nature (Dirac or Majorana) of massive neutrinos ν_j is one of the fundamental problems in the studies of neutrino mixing (see, e.g. [21]).

If neutrinos ν_j are Majorana fermions, processes in which the total lepton charge L is not conserved and changes by two units, such as $K^+ \rightarrow \pi^- + \mu^+ + \mu^+$, $\mu^+ + (A,Z) \rightarrow (A, Z+2) + \mu^-$, etc., should exist [5]. The only feasible experiments that at present have the potential of establishing the Majorana nature of massive neutrinos are the experiments searching for the neutrinoless double beta ($(\beta\beta)_{0\nu}$)-decay $(A,Z) \rightarrow (A, Z+2) + e^- + e^-$ (see, e.g. [42, 47, 48]). Under the assumption of $(\beta\beta)_{0\nu}$-decay generated *only by the (V-A) charged current weak interaction via the exchange of the three Majorana neutrinos* ν_j ($m_j \lesssim 1$ eV), the dependence of the $(\beta\beta)_{0\nu}$-decay amplitude $A(\beta\beta)_{0\nu}$ on the neutrino mass and mixing parameters factorizes in the effective Majorana mass $<\!m\!>$ (see, e.g. [42, 49]):

$$A(\beta\beta)_{0\nu} \sim\ <\!m\!>\ \mathcal{M}\ , \tag{3}$$

where $\mathcal{M}$ is the corresponding nuclear matrix element (NME) and $|<\!m\!>|$ is given by

$$|<\!m\!>| = \left| m_1 |U_{\rm e1}|^2 + m_2 |U_{\rm e2}|^2\ e^{i\alpha_{21}} + m_3 |U_{\rm e3}|^2\ e^{i\alpha_{31}} \right|\ . \tag{4}$$

If CP-invariance holds [6], one has [50] $\alpha_{21} = k\pi$, $\alpha_{31} = k'\pi$, where $k, k' = 0, 1, 2, ...$, and

$$\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1, \quad \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1 \tag{5}$$

represent the relative CP-parities of Majorana neutrinos ν_1 and ν_2, and ν_1 and ν_3, respectively. As eq. (3) indicates, the observation of $(\beta\beta)_{0\nu}$-decay of a given nucleus, and the measurement of the corresponding half-life, would allow a determination of $|<\!m\!>|$ only if the value of the relevant NME is known with a relatively small uncertainty.

The experimental searches for $(\beta\beta)_{0\nu}$-decay have a long history (see, e.g. [47, 49]). The best sensitivity was achieved in the Heidelberg–Moscow ^{76}Ge experiment [51]:

$$|<\!m\!>| < (0.35 - 1.05)\ \text{eV}, \quad \text{at 90\% C.L.}, \tag{6}$$

[4] The phases $\alpha_{21,31}$ can significantly affect the predictions for the rates of (LFV) decays $\mu \rightarrow e + \gamma$, $\tau \rightarrow \mu + \gamma$, etc. in a large class of supersymmetric theories with see-saw mechanism of ν-mass generation (see, e.g. [37]). Majorana CPV phases might be at the origin of baryon asymmetry of the Universe [38].

[5] The existing experimental constraints on the $|\Delta L| = 2$ processes have been discussed recently in, e.g. [46].

[6] We assume that $m_j > 0$ and that the fields of the Majorana neutrinos ν_j satisfy the Majorana condition: $C(\bar{\nu}_j)^T = \nu_j$, $j = 1, 2, 3$, where C is the charge conjugation matrix.

where a factor of 3 uncertainty associated with the calculation of the relevant nuclear matrix element [49] is taken into account. A similar result has been obtained by the IGEX collaboration [52]: $|<m>| < (0.33\text{–}1.35)$ eV (90% C.L.). A positive signal at $> 3\sigma$, corresponding to $|<m>| = (0.1\text{–}0.9)$ eV at 99.73% C.L., is claimed to be observed in [53]. This result will be checked in the currently running and future $(\beta\beta)_{0\nu}$-decay experiments. Two experiments, NEMO3 (with ^{100}Mo and ^{82}Se) [54] and CUORICINO (with ^{130}Te) [55], designed to reach a sensitivity of $|<m>| \sim(0.2\text{–}0.3)$ eV, are taking data. Their first results read (90% C.L.):

$$|<m>| < (0.7\text{–}1.2) \text{ eV} \;\; [54], \qquad |<m>| < (0.2\text{–}1.1) \text{ eV} \;\; [55], \tag{7}$$

where the estimated uncertainties in the NME are accounted for. A number of projects aim to reach a sensitivity to $|<m>| \sim (0.01\text{–}0.05)$ eV [48]: CUORE (^{130}Te), GERDA (^{76}Ge), EXO (^{136}Xe), MAJORANA (^{76}Ge), MOON (^{100}Mo), XMASS (^{136}Xe), CANDLES (^{48}Ca), SuperNEMO, etc. These experiments, in particular, can test the positive result claimed in [53] and probe the region of values of $|<m>|$ predicted in the case of IH and QD spectra [25].

In the present article we reanalyze the potential contribution that the future planned $(\beta\beta)_{0\nu}$-decay experiments can make to the studies of neutrino mixing. The observation of $(\beta\beta)_{0\nu}$-decay and the measurement of the corresponding half-life with a sufficient accuracy, would not only be a proof that the total lepton charge is not conserved in nature (see, e.g. [56]), but might provide also unique information i) on the type and possible hierarchical structure (NH, IH, QD, etc.) of the neutrino mass spectrum [19,25], ii) on the absolute scale of neutrino masses [58], and iii) on the Majorana CP-violation phases [24]. We consider 3-ν mixing, assume massive Majorana neutrinos and $(\beta\beta)_{0\nu}$-decay generated only by the $(V - A)$ charged current weak interaction via the exchange of the three Majorana neutrinos. As input in the analysis we use the results of recent studies of the precision that can be achieved in the measurement of the solar neutrino and CHOOZ mixing angles θ_{12} and θ_{13}, and of the neutrino mass squared differences Δm^2_{21} and $|\Delta m^2_{31}|$, on which $|<m>|$ depends. The uncertainty in the measured value of $|<m>|$, which is due to the imprecise knowledge of the relevant nuclear matrix elements, is also taken into account. All relevant errors are treated in a statistically self-consistent manner.

Our work is a continuation of earlier studies [7] (see, e.g. [19,25,41,58–63]). It is stimulated by the remarkable progress recently made in the experimental studies of ν-oscillations [3, 5, 10, 11] and by the recent analyses [26–30, 64, 65] in which the prospects for high precision determination of $\sin^2\theta_{12}$, $\sin^2\theta_{13}$, Δm^2_{21}, and $|\Delta m^2_{31}|$ in future ν-oscillation experiments have been extensively investigated. As a result of these studies the experiments that can provide the most precise measurement of the ν-oscillation parameters $|<m>|$ depends on, have been identified and a rather thorough evaluation of the precision that can be achieved has been made. In view of these developments a re-examination of the physics potential of the future $(\beta\beta)_{0\nu}$-decay experiments is both necessary and timely.

The outline of the paper is as follows. In Section 2 we briefly discuss the present status of the determination of, and the prospect for improvements of the precision on, the neutrino oscillation parameters relevant to the analysis of $(\beta\beta)_{0\nu}$-decay experiments. In Section 3 we review the predictions for the effective Majorana mass $|<m>|$ as a function of the lightest

[7] For an extensive list of references see, e.g. [57].

neutrino mass and the type of the neutrino mass spectrum, taking into account present and prospective uncertainties in the neutrino oscillation parameters. In Section 4 we present the results of a quantitative investigation of the potential of a future $(\beta\beta)_{0\nu}$-decay experiment based on a χ^2-analysis. We show our results as a function of quantities such as the observed mean value of $|<m>|$, its experimental uncertainty, and the uncertainty in the NME. We evaluate the possibility to obtain information on the lightest neutrino mass, the type of ν-mass spectrum and Majorana CPV phases from a $(\beta\beta)_{0\nu}$-decay experiment. In the latter case we take into account the constraint on the sum of neutrino masses Σ, which could be provided by cosmological observations, and investigate in detail the accuracies on $|<m>|$ and Σ, required in order to probe Majorana CP-violation in the lepton sector. Finally we conclude in Section 5.

2 The Neutrino Mixing Parameters and $|<m>|$

One can express [66] the two larger neutrino masses in terms of the lightest one, $\min(m_j) \equiv m_0 \equiv m_{\rm MIN}$, and of $\Delta m^2_\odot$ and $\Delta m^2_{\rm A}$ [8]. Within the convention we use, in both cases of ν-mass spectrum with normal and inverted hierarchy, one has: $\Delta m^2_\odot = \Delta m^2_{21} > 0$. For normal hierarchy, $\min(m_j) = m_1$, $\Delta m^2_{\rm A} = \Delta m^2_{31} > 0$, $m_2 = (m_1^2 + \Delta m^2_\odot)^{\frac{1}{2}}$, and $m_3 = (m_1^2 + \Delta m^2_{\rm A})^{\frac{1}{2}}$. If the spectrum is with inverted hierarchy, $\min(m_j) = m_3$, $\Delta m^2_{\rm A} = \Delta m^2_{32} < 0$ and thus $m_1 = (m_3^2 + |\Delta m^2_{\rm A}| - \Delta m^2_\odot)^{\frac{1}{2}} \cong (m_3^2 + |\Delta m^2_{\rm A}|)^{\frac{1}{2}}$, $m_2 = (m_3^2 + |\Delta m^2_{\rm A}|)^{\frac{1}{2}}$. For both types of mass ordering, the following relations hold: $|U_{\rm e1}|^2 = \cos^2\theta_\odot(1 - \sin^2\theta_{13})$, $|U_{\rm e2}|^2 = \sin^2\theta_\odot(1 - \sin^2\theta_{13})$, and $|U_{\rm e3}|^2 \equiv \sin^2\theta_{13}$, $\theta_\odot \equiv \theta_{12}$. Thus, in the case of interest the effective Majorana mass $|<m>|$, eq. (4), depends in general on: i) $\Delta m^2_{\rm A} = \Delta m^2_{31(32)}$, ii) $\theta_\odot = \theta_{12}$ and $\Delta m^2_\odot = \Delta m^2_{21}$, iii) the lightest neutrino mass m_0, iv) the mixing angle θ_{13}, and v) the Majorana CPV phases $\alpha_{21,31}$.

The best fit value and the 95% C.L. allowed range of $|\Delta m^2_{\rm A}|$ found in a combined analysis of the atmospheric neutrino [9] and K2K data read [5,14]:

$$
\begin{aligned}
|\Delta m^2_{\rm A}| &= 2.2 \times 10^{-3}\ {\rm eV}^2\ , \\
|\Delta m^2_{\rm A}| &= (1.7 - 2.9) \times 10^{-3}\ {\rm eV}^2\ .
\end{aligned}
\tag{8}
$$

Combined 2-ν oscillation analyses of the solar neutrino and KL 766.3 tyr spectrum data show [10,22] that $\Delta m^2_\odot$ and $\theta_\odot$ lie in the low-LMA region: $\Delta m^2_\odot = (7.9^{+0.6}_{-0.5}) \times 10^{-5}$ eV^2, $\tan^2\theta_\odot = (0.40^{+0.09}_{-0.07})$. The high-LMA solution (see, e.g. [67]) is excluded at $\sim 3.3\sigma$. Maximal solar neutrino mixing is ruled out at $\sim 6\sigma$; at 95% C.L. one finds $\cos 2\theta_\odot \geq 0.28$ [22], which has important implications for $|<m>|$ (see further). In the case of 3-ν mixing, the ν_e and $\bar{\nu}_e$ survival probabilities, relevant to the interpretation of the solar neutrino, KL and CHOOZ data, depend also on θ_{13} [21,68]. A combined 3-ν oscillation analysis of these data gives [14,22,23]

$$
\sin^2\theta_{13} < 0.027\ (0.047), \quad \text{at } 95\%\ (99.73\%)\ \text{C.L.} \tag{9}
$$

[8]For a discussion of the relevant formalism see, e.g. [19,57]. Notice that in [19] m_0 was used in the case of QD spectrum to indicate $m_0 \equiv m_1 \simeq m_2 \simeq m_3$. Here we extend this notation to indicate the smallest neutrino mass for each type of spectrum.

[9]The current atmospheric neutrino data are insensitive to θ_{13} satisfying the CHOOZ limit [5].

Furthermore, such an analysis shows [22] that for $\sin^2\theta_{13} \lesssim 0.02$ the allowed ranges of Δm^2_{21} and $\sin^2\theta_{21}$ do not differ substantially from those derived in the 2-ν oscillation analyses, and that the best fit values are practically independent of $\sin^2\theta_{13} < 0.05$. The best fit values and the allowed ranges at 95% C.L. read [14, 22]:

$$\begin{aligned} \Delta m^2_{21} &= 8.0 \times 10^{-5}~\mathrm{eV}^2, \quad \sin^2\theta_{21} = 0.31~, \\ \Delta m^2_{21} &= (7.3 - 8.5) \times 10^{-5}~\mathrm{eV}^2, \quad \sin^2\theta_{12} = (0.26 - 0.36)~. \end{aligned} \tag{10}$$

Existing data allow a determination of $\Delta m^2_{\odot}$, $\sin^2\theta_{\odot}$ and $|\Delta m^2_{\rm A}|$ at 3σ with an error of approximately 12%, 24%, and 50%, respectively. These parameters can (and very likely will) be measured with much higher accuracy in the future. The data from phase-III of the SNO experiment [3] [10] could lead to a reduction of the error in $\sin^2\theta_{12}$ to 21% [29, 30]. If instead of 766.3 tyr one uses simulated 3 ktyr KamLAND data in the same global solar and reactor neutrino data analysis, the 3σ errors in Δm^2_{21} and $\sin^2\theta_{12}$ diminish to 7% and 18% [30]. The most precise measurement of Δm^2_{21} could be achieved [29] using Super-Kamiokande doped with 0.1% of gadolinium for detection of reactor $\bar{\nu}_e$ [64]: the SK detector gets the same flux of reactor $\bar{\nu}_e$ as KamLAND and after 3 years of data-taking, Δm^2_{21} could be determined with an error of 3.5% at 3σ [29]. A dedicated reactor $\bar{\nu}_e$ experiment with a baseline $L \sim 60$ km, tuned to the minimum of the $\bar{\nu}_e$ survival probability, could provide the most precise determination of $\sin^2\theta_{12}$ [30, 65]: with statistics of ~ 60 GW ktyr and systematic error of 2% (5%), $\sin^2\theta_{12}$ could be measured with an error of 6% (9%) at 3σ [30]. The inclusion of the uncertainty in θ_{13} ($\sin^2\theta_{13}$ <0.05) in the analyses increases the quoted errors by (1–3)% to approximately 9% (12%) [30]. The highest precision in the determination of $|\Delta m^2_{\rm A}| = |\Delta m^2_{31}|$ is expected to be achieved from the studies of ν_μ-oscillations in the T2K (SK) [69] experiment: if the true $|\Delta m^2_{31}| = 2 \times 10^{-3}$ eV^2 (and true $\sin^2\theta_{23} = 0.5$), the 3σ uncertainty in $|\Delta m^2_{\rm A}|$ is estimated to be reduced in this experiment to $\sim$ 12% [26]. The error diminishes with increasing $|\Delta m^2_{31}|$.

In what regards the CHOOZ angle θ_{13}, there are several proposals for reactor $\bar{\nu}_e$ experiments with baseline $L \sim$ (1–2) km [27], which could improve the current limit, $\sin^2\theta_{13} < 0.05$, by a factor of (5–10): Double-CHOOZ (in France), Braidwood (in the USA), Daya-Bay (USA–China), KASKA (in Japan), etc. The reactor θ_{13} experiments can compete in sensitivity with accelerator experiments (T2K [69], NOνA [70]) (see, e.g. [26]) and can be done on a relatively short (for experiments in this field) time scale.

Information on the absolute scale of neutrino masses can be derived in ^{3}H β-decay experiments [71–73] and from cosmological and astrophysical data. The most stringent upper bounds on the $\bar{\nu}_e$ mass were obtained in the Troitzk [72] and Mainz [73] experiments:

$$m_{\bar{\nu}_e} < 2.3~\mathrm{eV} \quad \text{at 95\% C.L.}. \tag{11}$$

We have $m_{\bar{\nu}_e} \cong m_{1,2,3}$ in the case of the QD ν-mass spectrum. The KATRIN experiment [73] is planned to reach a sensitivity of $m_{\bar{\nu}_e} \sim 0.20$ eV, i.e. it will probe the region of the QD spectrum. The Cosmic Microwave Background (CMB) data of the WMAP experiment, combined with data from large scale structure surveys (2dFGRS, SDSS), lead to an upper limit on the sum of the neutrino masses [74]:

$$\sum_j m_j \equiv \Sigma < (0.7\text{–}2.0)~\mathrm{eV} \quad \text{at 95\% C.L.}, \tag{12}$$

[10] During this phase the neutral current rate will be measured in SNO with ^{3}He proportional counters.

where we have included a conservative estimate of the uncertainty in the upper limit (see, e.g. [75]). The WMAP and future PLANCK experiments can be sensitive to $\Sigma \cong 0.4$ eV. Data on weak lensing of galaxies by large scale structure, combined with data from the WMAP and PLANCK experiments, may allow Σ to be determined with an uncertainty of $\delta \sim$ (0.04–0.10) eV [76]. Similar sensitivities can be reached by analysing the distortions in the Cosmic Microwave Background due to gravitational lensing in a future high sensitivity experiment [77].

3 Predictions for the Effective Majorana Mass $|<m>|$

Given $\Delta m^2_\odot$, $|\Delta m^2_{\rm A}|$, $\theta_\odot$ and $\sin^2\theta_{13}$, the value of $|<m>|$ depends strongly on the type of the neutrino mass spectrum (NH, IH, QD, etc.) and on the Majorana CPV phases of the PMNS matrix, $\alpha_{21,31}$ (see eq. (4)). In what follows we will summarise the current status of the predictions for $|<m>|$.

Normal Hierarchical Spectrum. In this case $m_1 \ll m_2 \ll m_3$, $m_0 = m_1$, and one has [19]

$$\begin{aligned} |<m>| &= \left| \left(m_1 \cos^2\theta_\odot + e^{i\alpha_{21}} \sqrt{\Delta m^2_\odot + m_1^2}\, \sin^2\theta_\odot \right) \cos^2\theta_{13} \right. \\ &\quad \left. + \sqrt{\Delta m^2_{\rm A} + m_1^2}\, \sin^2\theta_{13}\, e^{i\alpha_{31}} \right|, \qquad (13) \\ &\simeq \left| \sqrt{\Delta m^2_\odot}\, \sin^2\theta_\odot \cos^2\theta_{13} + \sqrt{\Delta m^2_{\rm A}}\, \sin^2\theta_{13} e^{i(\alpha_{31}-\alpha_{21})} \right|, \qquad (14) \end{aligned}$$

where we have neglected m_1 in eq. (14). Although one neutrino, ν_1, effectively "decouples" and does not contribute to $|<m>|$, the value of $|<m>|$ according to eq. (14) still depends on the Majorana CPV phase difference $\alpha_{32} = \alpha_{31} - \alpha_{21}$. This reflects the fact that in contrast to the case of massive Dirac neutrinos (or quarks), CP-violation can take place in the mixing of only two massive Majorana neutrinos [17].

Since at 95% (99.73%) C.L. we have [14, 22] $\sqrt{\Delta m^2_\odot} \lesssim 9.2\ (9.4) \times 10^{-3}$ eV, $\sin^2\theta_\odot \lesssim 0.36\ (0.40)$, $\sqrt{\Delta m^2_{\rm A}} \lesssim 5.4\ (5.7) \times 10^{-2}$ eV, $\sin^2\theta_{13} < 0.027\ (0.046)$, and the largest neutrino mass enters into the expression for $|<m>|$ multiplied by the factor $\sin^2\theta_{13}$, the predicted value of $|<m>|$ is typically $\sim$ few$\times 10^{-3}$ eV: for $\sin^2\theta_{13} = 0.03$, one finds $|<m>| \lesssim 0.005$ eV (using the data at 95% C.L.). Using the best fit values of the indicated parameters (see eqs. (8) and (10)) and the same value of $\sin^2\theta_{13} = 0.03$, we get $|<m>| \lesssim 0.0042$ eV.

The minimal value of $|<m>|$ in eq. (14) is obtained when there is a maximal compensation between the "solar neutrino" term, $\sqrt{\Delta m^2_\odot}\sin^2\theta_\odot\cos^2\theta_{13}$, and the "atmospheric neutrino" one, $\sqrt{\Delta m^2_{\rm A}}\sin^2\theta_{13}$. At 95% (99.73%) C.L. we have $\sqrt{\Delta m^2_{\rm A}}\sin^2\theta_{13} \lesssim 1.5\ (2.7) \times 10^{-3}$ eV, while the "solar neutrino" term takes values in the interval $(2.1 - 3.2) \times 10^{-3}$ eV $((1.9 - 3.6) \times 10^{-3}$ eV). Thus, at 95% C.L. the "solar neutrino" term is larger than the "atmospheric neutrino" one and $|<m>|$ is bounded from below. However, this may not be true considering the current 99.73% C.L. intervals of allowed values of the relevant oscillation parameters.

It follows from eq. (13) and the allowed ranges of values of $\Delta m^2_\odot$, $\Delta m^2_{\rm A}$, $\sin^2\theta_\odot$, $\sin^2\theta_{13}$, as well as of the lightest neutrino mass m_1 and CPV phases $\alpha_{21,31}$, that in the case of spectrum with *normal hierarchy* there can be a complete cancellation between the three terms in eq. (13), and one can have [58] $|<\!m\!>| = 0$.

Inverted Hierarchical Spectrum. For IH neutrino mass spectrum, $\Delta m^2_{\rm A} < 0$, $m_0 = m_3$, and $m_3 \ll m_1 \cong m_2 \cong \sqrt{|\Delta m^2_{\rm A}|} = \sqrt{\Delta m^2_{23}}$. Using eq. (4) we find [19, 24]:

$$
\begin{aligned}
|<\!m\!>| &\cong \left| (\cos^2\theta_\odot + e^{i\alpha_{21}}\sin^2\theta_\odot)\cos^2\theta_{13}\sqrt{m_3^2 + |\Delta m^2_{\rm A}|} + m_3\sin^2\theta_{13}\, e^{i\alpha_{31}} \right|, \quad (15)\\
&\cong \sqrt{m_3^2 + |\Delta m^2_{\rm A}|}\cos^2\theta_{13}\left(1 - \sin^2 2\theta_\odot \sin^2\frac{\alpha_{21}}{2}\right)^{\frac{1}{2}}, \quad (16)\\
&\cong \sqrt{|\Delta m^2_{\rm A}|}\cos^2\theta_{13}\left(1 - \sin^2 2\theta_\odot \sin^2\frac{\alpha_{21}}{2}\right)^{\frac{1}{2}}, \quad (17)
\end{aligned}
$$

where we have neglected $m_3\sin^2\theta_{13}$ in eqs. (16) and (17). The term $m_3\sin^2\theta_{13}$ can always be neglected given the existing data: even in the case where the spectrum is with *partial* inverted hierarchy and $m_3^2 \sim |\Delta m^2_{\rm A}|$, the minimum of the sum of the other two terms in $|<\!m\!>|$ satisfies $\sqrt{m_3^2 + |\Delta m^2_{\rm A}|}\cos 2\theta_\odot \cos^2\theta_{13} \gg m_3\sin^2\theta_{13}$, since the data on $\theta_\odot$ and θ_{13} imply $\cos 2\theta_\odot \gg \tan^2\theta_{13}$. Even though in eqs. (16) and (17) one of the three massive Majorana neutrinos "decouples", the value of $|<\!m\!>|$ depends on the Majorana CP-violating phase α_{21}. It follows from eq. (17) that

$$
\sqrt{|\Delta m^2_{\rm A}|}\ \cos 2\theta_\odot\ \cos^2\theta_{13} \le\ |<\!m\!>| \le \sqrt{|\Delta m^2_{\rm A}|}\cos^2\theta_{13}. \qquad (18)
$$

The lower and upper limits correspond to the CP-conserving cases $\alpha_{21} = \pi$ and $\alpha_{21} = 0$. Most remarkably, since according to the solar neutrino and KamLAND data $\cos 2\theta_\odot \sim 0.40$ and $\cos 2\theta_\odot \gtrsim 0.28$ at 95% C.L., we get a significant lower limit on $|<\!m\!>|$ exceeding 10^{-2} eV in this case [25, 58]. Using, e.g. the best fit values of $|\Delta m^2_{\rm A}|$ and $\sin^2\theta_\odot$ we find: $|<\!m\!>| \gtrsim 0.02$ eV. The maximal value of $|<\!m\!>|$ is determined by $|\Delta m^2_{\rm A}|$ and, according to eqs. (8) and (9), can reach $|<\!m\!>| \sim 0.055$ eV. The indicated values of $|<\!m\!>|$ are within the range of sensitivity of the next generation of $(\beta\beta)_{0\nu}$-decay experiments.

Let us note that if $\Delta m^2_{\rm A} < 0$, i.e. if the neutrino mass spectrum is with inverted hierarchy, an upper limit on $\Sigma = (m_1 + m_2 + m_3) \le 0.10$ eV would imply $m_3 \lesssim 0.02$ eV and therefore $m_3^2 \ll |\Delta m^2_{\rm A}|$. In this case the spectrum would be of the IH type and eqs. (17) and (18) would be valid.

The expression for $|<\!m\!>|$, eq. (17), permits to relate the value of $\sin^2\alpha_{21}/2$ to the experimentally measurable quantities [19, 24] $|<\!m\!>|$, $\Delta m^2_{\rm A}$ and $\sin^2 2\theta_\odot$:

$$
\sin^2\frac{\alpha_{21}}{2} \cong \left(1 - \frac{|<\!m\!>|^2}{|\Delta m^2_{\rm A}|\cos^4\theta_{13}}\right)\frac{1}{\sin^2 2\theta_\odot}\ . \qquad (19)
$$

A sufficiently accurate measurement of $|<\!m\!>|$ and of $|\Delta m^2_{\rm A}|$ and $\theta_\odot$, could allow us to get information about the value of α_{21}. If, e.g. the data show unambiguously that $|<\!m\!>| < \sqrt{|\Delta m^2_{\rm A}|}\cos^2\theta_{13}$, that would imply $\alpha_{21} \neq 0$. If in addition the data show that $|<\!m\!>| > \sqrt{|\Delta m^2_{\rm A}|}\cos 2\theta_\odot\cos^2\theta_{13}$, one could conclude that α_{21} takes a CP-violating value.

Three Quasi-Degenerate Neutrinos. In this case $m_0 \equiv m_1 \cong m_2 \cong m_3$, $m_0^2 \gg |\Delta m^2_{\rm A}|$, $m_0 \gtrsim 0.10$ eV. Hence, $|\!<\!m\!>\!|$ is essentially independent of $\Delta m^2_{\rm A}$ and $\Delta m^2_{\odot}$, and the two possibilities, $\Delta m^2_{\rm A} > 0$ and $\Delta m^2_{\rm A} < 0$, lead *effectively* to the same predictions for $|\!<\!m\!>\!|$. The mass m_0 coincides with the $\bar{\nu}_e$ mass $m_{\bar{\nu}_e}$ measured in the ^{3}H β-decay experiments: $m_0 = m_{\bar{\nu}_e}$. Thus, $m_0 < 2.3$ eV, or if we use a conservative cosmological upper limit [75], $m_0 = \Sigma/3 < 0.7$ eV. The QD ν-mass spectrum is realized for values of m_0, that can be measured in the ^{3}H β-decay experiment KATRIN [73]. The effective Majorana mass $|\!<\!m\!>\!|$ is given by

$$|\!<\!m\!>\!| \;\cong\; m_0 \left|(\cos^2\theta_{\odot} + \;\sin^2\theta_{\odot} e^{i\alpha_{21}})\;\cos^2\theta_{13} + e^{i\alpha_{31}}\sin^2\theta_{13}\right|, \tag{20}$$

$$\cong\; m_0\;\left|\cos^2\theta_{\odot} + \;\sin^2\theta_{\odot} e^{i\alpha_{21}}\right| = m_0\;\sqrt{1 - \sin^2 2\theta_{\odot}\sin^2\frac{\alpha_{21}}{2}}. \tag{21}$$

Similarly to the case of the IH spectrum, one has:

$$m_0\;\cos 2\theta_{\odot} \lesssim |\!<\!m\!>\!| \leq m_0\;. \tag{22}$$

For $\cos 2\theta_{\odot} \sim 0.40$, favored by the data, one finds a non-trivial lower limit on $|\!<\!m\!>\!|$, $|\!<\!m\!>\!| \gtrsim 0.08$ eV. For values of the parameters allowed at 95% C.L. one has $|\!<\!m\!>\!| \gtrsim 0.06$ eV. Using the conservative cosmological upper bound on Σ we get $|\!<\!m\!>\!| < 0.70$ eV. Also in this case one can obtain, in principle, direct information on one CPV phase from the measurement of $|\!<\!m\!>\!|$, m_0 and $\sin^2 2\theta_{\odot}$:

$$\sin^2\frac{\alpha_{21}}{2} \cong \left(1 - \frac{|\!<\!m\!>\!|^2}{m_0^2}\right)\frac{1}{\sin^2 2\theta_{\odot}}. \tag{23}$$

The specific features of the predictions for $|\!<\!m\!>\!|$ in the cases of the three types of neutrino mass spectrum discussed above are evident in Fig. 1, where the dependence of $|\!<\!m\!>\!|$ on $m_0 = \min(m_j)$ for $\sin^2\theta_{\odot} = 0.31$ and $\sin^2\theta_{13} = 0.01$ and 0.03 is shown. The figures are obtained by including a 2σ uncertainty in the predicted value of $|\!<\!m\!>\!|$. The uncertainty in $|\!<\!m\!>\!|$, $\sigma(|\!<\!m\!>\!|)$, has been calculated by exploiting the explicit dependence of $|\!<\!m\!>\!|$ on the oscillation parameters $\Delta m^2_{\odot}$, $\Delta m^2_{\rm A}$, $\sin^2\theta_{\odot}$ and $\sin^2\theta_{13}$ and assuming the following 1σ errors (achievable in the future) in the determination of the latter: $\sigma(\Delta m^2_{\odot}) = 2\%$, $\sigma(|\Delta m^2_{\rm A}|) = 6\%$, $\sigma(\sin^2\theta_{12}) = 4\%$ and two values of $\sigma(\sin^2\theta_{13}) = 0.004$ and 0.008. The current best fit values of $\Delta m^2_{\odot}$ and $|\Delta m^2_{\rm A}|$ have been used. The Majorana CP-violation phases α_{21} and α_{31} were varied over all possible values they can take [11]. For the NH and QD (and interpolating) spectra, the regions within the black lines of a given type (solid, short-dashed, long-dashed, dash-dotted) correspond to the four different sets of CP-conserving values of the two phases α_{21} and α_{31}, and thus to the four possible combinations of the relative CP parities (η_{21},η_{31}) of neutrinos $\nu_{1,2}$ and $\nu_{1,3}$: $(+1,+1)$ solid, $(-1,-1)$ short-dashed, $(+1,-1)$ long-dashed, and $(-1,+1)$ dash-dotted lines. If the spectrum is IH, the contribution to $|\!<\!m\!>\!|$ due to m_3 can be neglected and the predictions for $|\!<\!m\!>\!|$ become practically independent of α_{31} (η_{31}). In this case the regions delimited by the black solid (dotted) lines correspond to $\eta_{21} = +1$ ($\eta_{21} = -1$). In the case of CP-violation all colored regions are allowed.

[11] It follows from eq. (4) that $|\!<\!m\!>\!|$ is symmetric under the transformations $\alpha_{21,31} \rightarrow 2\pi - \alpha_{21,31}$. This implies that it is sufficient to consider values of $\alpha_{21,31}$ in the range $[0,\pi]$ to cover all possible physical configurations for $|\!<\!m\!>\!|$.

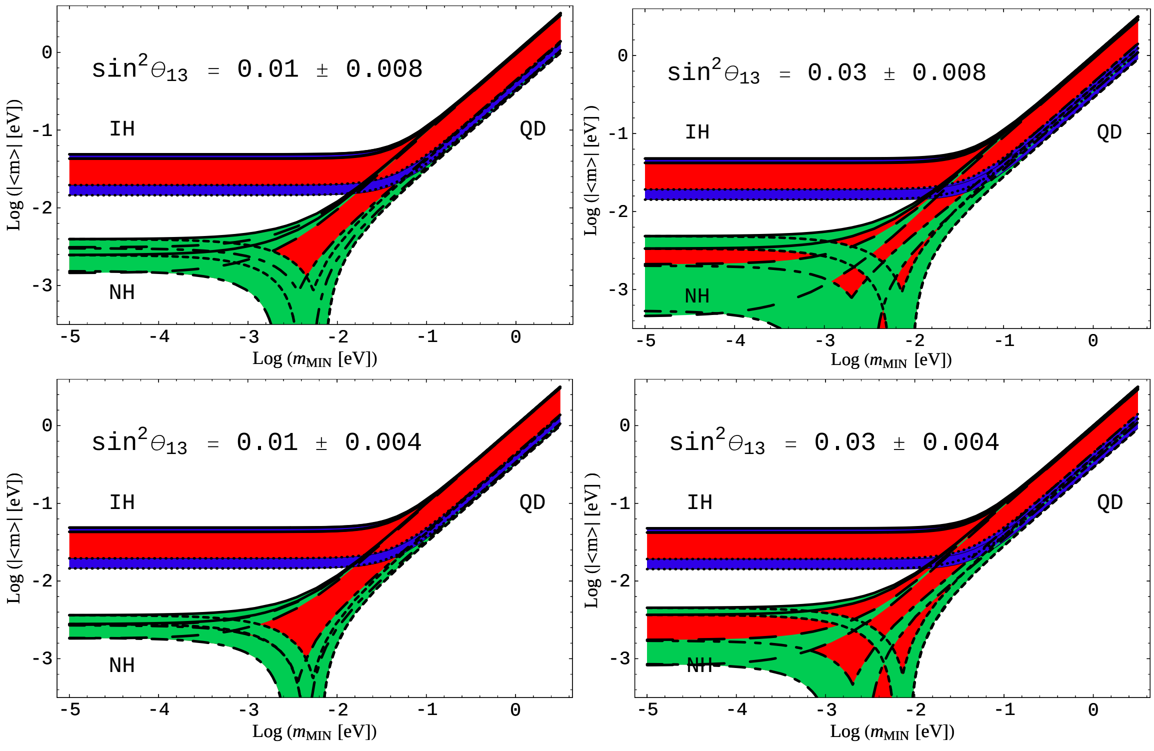

Figure 1: The predicted value of $|\!<\!m\!>\!|$ (including a prospective 2σ uncertainty) as a function of $\min(m_j)$ for $\sin^2\theta_\odot = 0.31$ and $\sin^2\theta_{13} = 0.01;\ 0.03$, and two different assumptions on the error on $\sin^2\theta_{13}$. For the NH and QD (and interpolating) spectra, the regions within the black lines of a given type (solid, short-dashed, long-dashed, dash-dotted) correspond to the four different sets of CP-conserving values of the two phases α_{21} and α_{31}, and thus to the four possible combinations of the relative CP parities (η_{21}, η_{31}) of neutrinos $\nu_{1,2}$ and $\nu_{1,3}$: $(+1,+1)$ solid, $(-1,-1)$ short-dashed, $(+1,-1)$ long-dashed, and $(-1,+1)$ dash-dotted lines. For the IH spectrum, the regions delimited by the black solid (dotted) lines correspond to $\eta_{21} = +1$ ($\eta_{21} = -1$), independently of η_{31}. The regions shown in red/medium-gray correspond to violation of CP-symmetry (see text for further details).

The regions shown in red/medium-gray are the so-called "just CP-violation" regions [19]: an experimental point in these regions would unambiguously signal CP-violation associated with Majorana neutrinos. In the regions shown in blue/dark-gray and in green/light-gray it is not possible to distinguish between CP-violation and CP-conservation, because of the uncertainty implied by the errors on the oscillation parameters.

The impact the prospective errors on $\Delta m^2_{\rm A}$, $\Delta m^2_\odot$ and $\sin^2\theta_\odot$ have on the predictions for $|\!<\!m\!>\!|$ is, in general, very small. More specifically, in the case of QD spectrum, the contributions of $\sigma(\Delta m^2_{\rm A})$, $\sigma(\Delta m^2_\odot)$ and $\sigma(\sin^2\theta_{13})$ in $\sigma(|\!<\!m\!>\!|)$ can be neglected and only $\sigma(\sin^2\theta_\odot)$ induces an uncertainty in $|\!<\!m\!>\!|$ which can be as large as few % if $\alpha_{21} \sim \pi$. For the IH type of spectrum, both $\sigma(|\Delta m^2_{\rm A}|)$ and $\sigma(\sin^2\theta_{12})$ are relevant and contribute to an overall $\sigma(|\!<\!m\!>\!|) \sim$ several %. Also in this case the error on $|\!<\!m\!>\!|$ due to $\sigma(\sin^2\theta_{12})$ increases as α_{21} varies from 0 to π. For NH spectrum, the dominant source of error is $\sigma(\sin^2\theta_{13})$. It affects significantly the predicted value of $|\!<\!m\!>\!|$. This explains the different allowed ranges of values for $|\!<\!m\!>\!|$ obtained for $\sigma(\sin^2\theta_{13}) = 0.004$ and 0.008. Notice that the impact of the errors is larger the smaller $|<m>|$ is, i.e. when $(\alpha_{31} - \alpha_{21})$ approaches the value π.

If the spectrum is with normal hierarchy ($\Delta m^2_{\rm A} > 0$), $|<m>|$ can lie anywhere between 0 and the currently existing upper limits, eqs. (6) and (7). This conclusion does not change even under the most favorable conditions for the determination of $|<m>|$, namely, even when $|\Delta m^2_{\rm A}|$, $\Delta m^2_{\odot}$, $\theta_\odot$ and θ_{13} are known with negligible uncertainty.

The "gap" between the predicted values of $|<m>|$ in the cases of IH and NH spectra allows us, in principle, to distinguish between these two types of *hierarchical* spectra [25,58]. Establishing, for instance, that $|<m>| \neq 0$ but $|<m>| < 10^{-2}$ eV would imply, within the 3-neutrino mixing scheme with Majorana neutrinos under discussion, that the neutrino mass spectrum is with normal hierarchy, i.e. $\Delta m^2_{\rm A} > 0$. Depending on the value of m_1, the spectrum could be *either normal hierarchical (NH) or with partial hierarchy* [19]. Obviously, such a result would rule out both the IH and QD spectrum.

If the results in [53] implying $|<m>| = (0.1–0.9)$ eV are confirmed, this would mean, in particular, that the neutrino mass spectrum is of the QD type. In this case, however, the measurement of $|<m>|$ cannot provide information on the $\text{sgn}(\Delta m^2_{\rm A})$.

It should be clear from the preceding discussion that, depending on the measured value of $|<m>| \neq 0$, the $(\beta\beta)_{0\nu}$-decay experiments may or may not provide information on *both* the type of ν mass spectrum (NH, IH, QD, etc.) and $\text{sgn}(\Delta m^2_{\rm A})$. If $|<m>| \sim$ few $\times$ 10^{-3} eV $< 10^{-2}$ eV, both the type of the spectrum and $\text{sgn}(\Delta m^2_{\rm A})$ will be determined. For $\sqrt{|\Delta m^2_{\rm A}|}\ \cos 2\theta_\odot \leq\ |<m>| \leq \sqrt{|\Delta m^2_{\rm A}|}$, it would be possible to conclude that $\text{sgn}(\Delta m^2_{\rm A}) <$ 0 only if $m_0 \lesssim 0.02$ eV, i.e. $m_0^2 \ll |\Delta m^2_{\rm A}|$. In a relatively narrow interval of values of $m_0 \sim$ few $\times\ 10^{-2}$ eV, for which $m_0^2 \sim |\Delta m^2_{\rm A}|$, one can have both $\Delta m^2_{\rm A} < 0$ and $\Delta m^2_{\rm A} > 0$. In the latter case the ν mass spectrum is with *partial hierarchy.* If $|<m>| \gtrsim 0.10$ eV, the ν mass spectrum is QD and the measurement of $|<m>|$ will provide no information on $\text{sgn}(\Delta m^2_{\rm A})$.

Finally, if neutrino oscillation experiments show that $\Delta m^2_{\rm A} < 0$ and therefore the ν mass spectrum is with inverted hierarchy, while in $(\beta\beta)_{0\nu}$-decay experiments only the upper limit $|<m>| < \sqrt{|\Delta m^2_{\rm A}|}\cos 2\theta_\odot \cos^2\theta_{13}$ is obtained, that would mean either that there is a new additional contribution to the $(\beta\beta)_{0\nu}$-decay amplitude which interferes destructively with that due to the light Majorana neutrino exchange, or that the massive neutrinos ν_j are Dirac particles. Similar conclusion could be made if, e.g., the KATRIN experiment shows that $m_0 \gtrsim 0.2$ eV and correspondingly the ν mass spectrum is QD, while $(\beta\beta)_{0\nu}$-decay experiments demonstrate only that the upper limit $|<m>| < m_0 \cos 2\theta_\odot$ holds.

4 Analysis of the Implications of a $(\beta\beta)_{0\nu}$-Decay Half-Life Measurement

4.1 On the NME Uncertainties

If the $(\beta\beta)_{0\nu}$-decay of a given nucleus is observed, it will be possible to determine the value of $|<m>|$ from the measurement of the associated half-life of the decay. This would require the knowledge of the nuclear matrix element of the process. At present there exist large uncertainties in the calculation of the $(\beta\beta)_{0\nu}$-decay nuclear matrix elements (see, e.g. [49]). This is reflected, in particular, in the factor of ~ 3 uncertainty in the upper limit on $|{<}m{>}|$, which is extracted from the experimental lower limits on the $(\beta\beta)_{0\nu}$-decay half-life of ^{76}Ge.[12] For some nuclei (such as ^{100}Mo, ^{130}Te, ^{136}Xe), the uncertainties can be even larger. Recently, encouraging results on the problem of calculating the nuclear matrix elements have been obtained in [78]. A discussion of the problems related to the calculation of the $(\beta\beta)_{0\nu}$-decay NME is outside the scope of the present work. We would like to only note here that the observation of a $(\beta\beta)_{0\nu}$-decay of one nucleus is likely to lead to searches and eventually to observation of the decay of other nuclei. It can be expected that such a progress will help, in particular, to solve the problem of the sufficiently precise calculation of the nuclear matrix elements for the $(\beta\beta)_{0\nu}$-decay [79].

4.2 The Method of Analysis

The experimental observable in $(\beta\beta)_{0\nu}$-decay is the decay rate $\Gamma_{\rm obs}$ measured with an experimental accuracy $\sigma(\Gamma_{\rm obs})$. The observed decay rate has to be compared with the theoretically predicted rate

$$\Gamma_{\rm th} = G\,|\mathcal{M}|^2\,\left(|{<}m{>}|(\boldsymbol{x})\right)^2\,, \tag{24}$$

where G is a known phase space factor and $\mathcal{M}$ is the NME. In eq. (24) $\boldsymbol{x} = (\boldsymbol{x}_{\rm osc}, \boldsymbol{x}^{0\nu}_{\beta\beta})$ are the parameters determining $|{<}m{>}|$, which we divide into parameters measured in oscillation experiments, whose values we are going to use as input in the analysis,

$$\boldsymbol{x}_{\rm osc} = (\theta_{12}, \theta_{13}, |\Delta m^2_{31}|, \Delta m^2_{21})\,, \tag{25}$$

and parameters that are, in principle, accessible by $(\beta\beta)_{0\nu}$-decay experiments,

$$\boldsymbol{x}^{0\nu}_{\beta\beta} = (m_0, {\rm sgn}(\Delta m^2_{31}), \alpha_{21}, \alpha_{31})\,. \tag{26}$$

To investigate the potential to get information on the parameters $\boldsymbol{x}^{0\nu}_{\beta\beta}$ from the result of a generic $(\beta\beta)_{0\nu}$-decay experiment, we convert the observed decay rate and the experimental error into an "observed effective Majorana mass" and its error by

$$|{<}m{>}|^{\rm obs} \equiv \sqrt{\frac{\Gamma_{\rm obs}}{G}}\,\frac{1}{|\mathcal{M}_0|}\,, \quad \sigma_{\beta\beta} = \frac{1}{2}\,\frac{1}{\sqrt{\Gamma_{\rm obs}G}}\,\frac{1}{|\mathcal{M}_0|}\,\sigma(\Gamma_{\rm obs})\,, \tag{27}$$

where $|\mathcal{M}_0|$ is some nominal (theoretically predicted) value of the NME. If $|{<}m{>}|^{\rm obs} > n\sigma_{\beta\beta}$, a positive $(\beta\beta)_{0\nu}$-decay signal is observed at the $n\sigma$ C.L. Otherwise only an upper bound on $|{<}m{>}|$ is obtained. The quantity $\sigma_{\beta\beta}$ defined in eq. (27) is a measure for the "accuracy" of the experiment. Then we construct a χ^2 in the following way:

$$\chi^2(\boldsymbol{x}^{0\nu}_{\beta\beta}, F) = \min_{\xi\in[1/\sqrt{F},\sqrt{F}]} \frac{\left[\xi\,|{<}m{>}|(\boldsymbol{x}) - |{<}m{>}|^{\rm obs}\right]^2}{\sigma^2_{\beta\beta} + \xi^2\sigma^2_{\rm th}}\,. \tag{28}$$

[12] For the uncertainty on the NME for the $(\beta\beta)_{0\nu}$-decay half-life of ^{76}Ge commonly a factor of 10 is adopted. Since $|{<}m{>}|$ depends on the square-root of the half-life, typical values for the current uncertainty on $|{<}m{>}|$ are factors from 3 to 4.

The parameter ξ takes into account the uncertainty on the NME, and it is defined by

$$\xi \equiv \frac{|\mathcal{M}|}{|\mathcal{M}_0|}\,, \tag{29}$$

where $|\mathcal{M}|$ is the unknown *true* value of the NME and $|\mathcal{M}_0|$ is the nominal value used in eq. (27) to obtain $|{<}m{>}|^{\rm obs}$. The theoretical error $\sigma_{\rm th}$ in eq. (28) takes into account the uncertainty implied by the errors on the oscillation parameters; it is calculated by

$$\sigma^2_{\rm th} = \sigma^2_{\rm th}(\boldsymbol{x}_{\rm osc}) = \sum_i \left(\frac{\partial |{<}m{>}|}{\partial x^i_{\rm osc}}\right)^2 (\delta x^i_{\rm osc})^2\,, \tag{30}$$

where the index i runs over the four oscillation parameters given in eq. (25), $\delta x^i_{\rm osc}$ is the uncertainty on the parameter $x^i_{\rm osc}$, and we have used the fact that to very good approximation the errors on the oscillation parameters are uncorrelated (see, e.g. [14]).

Assuming that the value of the NME is known within a factor $F \geq 1$, for given parameters $\boldsymbol{x}_{\beta\beta}$ we minimise the right-hand side of eq. (28) with respect to ξ, allowing ξ to vary within the interval $[1/\sqrt{F}, \sqrt{F}]$. A perfectly known NME corresponds to $F = 1$. Note that in this way we do not introduce a probability weight for the NME; all values between $|\mathcal{M}_0|/\sqrt{F}$ and $\sqrt{F}|\mathcal{M}_0|$ are treated on an equal footing. This procedure is similar to the "flat priors" used in unitarity triangle fits of the CKM matrix in order to account for theoretical uncertainties, see e.g. [80]. We have adopted this method, since it is not possible to assign a well defined probability distribution to the parameter ξ, and therefore, specifying a range for ξ without imposing any further weight seems to be the most reliable procedure. Since the choice of the NME uncertainty factor F is subject to some arbitrariness we shall show results for various values of F.

To combine a measurement of the $(\beta\beta)_{0\nu}$-decay rate with a constraint on the sum of the neutrino masses Σ (obtained, e.g. from cosmological/astrophysical observations), we generalise eq. (28) in a straightforward way. To take into account the correlations between $|{<}m{>}|$ and Σ induced by the uncertainties on Δm^2_{21} and Δm^2_{31}, we use the following covariance matrix in the χ^2-analysis:

$$S_{ab} = \delta_{ab}(\sigma^{\rm exp}_a)^2 + \sum_i \frac{\partial T_a}{\partial x^i_{\rm osc}} \frac{\partial T_b}{\partial x^i_{\rm osc}} (\delta x^i_{\rm osc})^2\,, \quad a, b = 1, 2, \tag{31}$$

where $T_1 \equiv \xi |{<}m{>}|$, $T_2 \equiv \Sigma$, and $\sigma^{\rm exp}_1 \equiv \sigma_{\beta\beta}$ and $\sigma^{\rm exp}_2 \equiv \sigma_\Sigma$ are the experimental errors on $|{<}m{>}|^{\rm obs}$ and Σ, respectively.

4.3 Constraining the Lightest Neutrino Mass

We start the quantitative evaluation of the physics potential of a $(\beta\beta)_{0\nu}$-decay observation by discussing the information that can be obtained on the absolute value of the lightest neutrino mass m_0. Given an experimental result on $|{<}m{>}|$ from a $(\beta\beta)_{0\nu}$-decay experiment, one can infer an allowed range for m_0 for each type of neutrino mass ordering. The results of such an analysis are shown in Fig. 2. For given values of $|{<}m{>}|^{\rm obs}$ and its experimental error $\sigma_{\beta\beta}$, we minimize the χ^2 of eq. (28) with respect to the phases α_{21} and α_{31}, and calculate the allowed range for m_0 at 2σ by using the condition $\chi^2(m_0) \leq 4$. In Fig. 2 we adopted the best fit values for Δm^2_{21}, $|\Delta m^2_{31}|$ and $\sin^2\theta_{12}$ (see eqs. (8) and (10)), and $\sin^2\theta_{13} = 0$. We have verified that the results hardly change if the values of the oscillation parameters are varied within the present 3σ ranges. The dashed lines in Fig. 2 correspond to the current uncertainties of Δm^2_{21}, $|\Delta m^2_{31}|$, $\sin^2\theta_{12}$ and $\sin^2\theta_{13}$, while the solid lines are obtained using the following prospective 1σ errors: $\sigma(\Delta m^2_{21}) = 2\%$, $\sigma(|\Delta m^2_{31}|) = 5\%$, $\sigma(\sin^2\theta_{12}) = 3\%$ and $\sigma(\sin^2\theta_{13}) = 0.002$. By comparing the dashed and solid lines in Fig. 2 one observes that improving the accuracy of the oscillation parameters has only a minor impact on the results: we find only small improvements of the constraints on m_0, while the qualitative behavior is unchanged. Consider first the case of $|<m>|^{\rm obs} = 0.2$ eV, shown in the right column of Fig. 2. In this case a positive signal should be established with high confidence by the next generation of $(\beta\beta)_{0\nu}$-experiments. If the experimental error in $|<m>|^{\rm obs}$ is sufficiently small ($\sigma_{\beta\beta} \lesssim 0.06$ eV for NME uncertainty factor $F \leq 3$), i) the NH and IH spectra will be excluded and hence, the neutrino mass spectrum will be proved to be QD, ii) m_0 will be constrained to lie in a rather narrow interval of values limited from below by $m_0 \gtrsim 0.1$ eV, and iii) no information on $\text{sgn}(\Delta m^2_{31})$ will be obtained. The uncertainty in the NME directly translates into an uncertainty in m_0.

In the case of an "intermediate" value of $|<m>|^{\rm obs} = 0.04$ eV shown in the middle column of Fig. 2, a lower and an upper bound on m_0 can be established for $\Delta m^2_{31} > 0$ if $\sigma_{\beta\beta} \lesssim 0.017$ eV: $0.01 \text{ eV} \lesssim m_0 \lesssim 0.1$ eV. In the case of $\Delta m^2_{31} < 0$ only an upper bound will be obtained: $m_0 \lesssim 0.1$ eV. This result can be easily understood from Fig. 1: if $|<m>|$ is sufficiently large and $\sigma_{\beta\beta}$ is small enough, the branch corresponding to the *normal hierarchical* spectrum extending to $m_0 = 0$ can be excluded.

Consider finally the left column of plots in Fig. 2 corresponding to a very small value of $|<m>|^{\rm obs} = 4 \times 10^{-3}$ eV. For estimated typical values of $\sigma_{\beta\beta}$ of the next generation of $(\beta\beta)_{0\nu}$-decay experiments and the mean value of $|<m>|$ considered, only an upper bound on $|<m>|$ can be established. It is clear that in this case one gets also only an upper bound on m_0. Moreover, from the panel corresponding to a known NME ($F = 1$) one observes that for (ambitious) experimental accuracies, i.e. for $\sigma_{\beta\beta} \lesssim 7 \times 10^{-3}$ eV, the case of $\text{sgn}(\Delta m^2_{31}) < 0$ (inverted mass hierarchy) can, in principle, be excluded. This is a consequence of the lower bound on $|<m>|$ for inverted ordering, which follows from the fact that $\cos 2\theta_{12}$ is significantly different from zero (see eq. (18) and the related discussion). However, if we take into account a possible uncertainty in the NME, the requirements on the experimental accuracy of $|<m>|$ become exceedingly demanding ($\sigma_{\beta\beta} \lesssim 4 \times 10^{-3}$ eV for $F = 2$), which renders the exclusion of the neutrino mass spectrum with inverted hierarchy remarkably challenging. Reducing the error to $\sigma_{\beta\beta} \cong 10^{-3}$ eV would allow, e.g. for $F \leq 2$, to conclude that $m_0 \lesssim 0.02$ eV and the neutrino mass spectrum is *normal hierarchical*. Establishing in an independent experiment that $\text{sgn}(\Delta m^2_{31}) < 0$ would imply in the case under consideration that there are additional mechanism(s) of $(\beta\beta)_{0\nu}$-decay [81] whose contribution to the $(\beta\beta)_{0\nu}$-decay amplitude compensates partially the one due to the Majorana neutrino exchange.

4.4 Determining the Type of Neutrino Mass Spectrum

As is clear from the previous discussions, $(\beta\beta)_{0\nu}$-decay experiments provide a unique possibility to obtain information on the type of neutrino mass spectrum, i.e. to distinguish

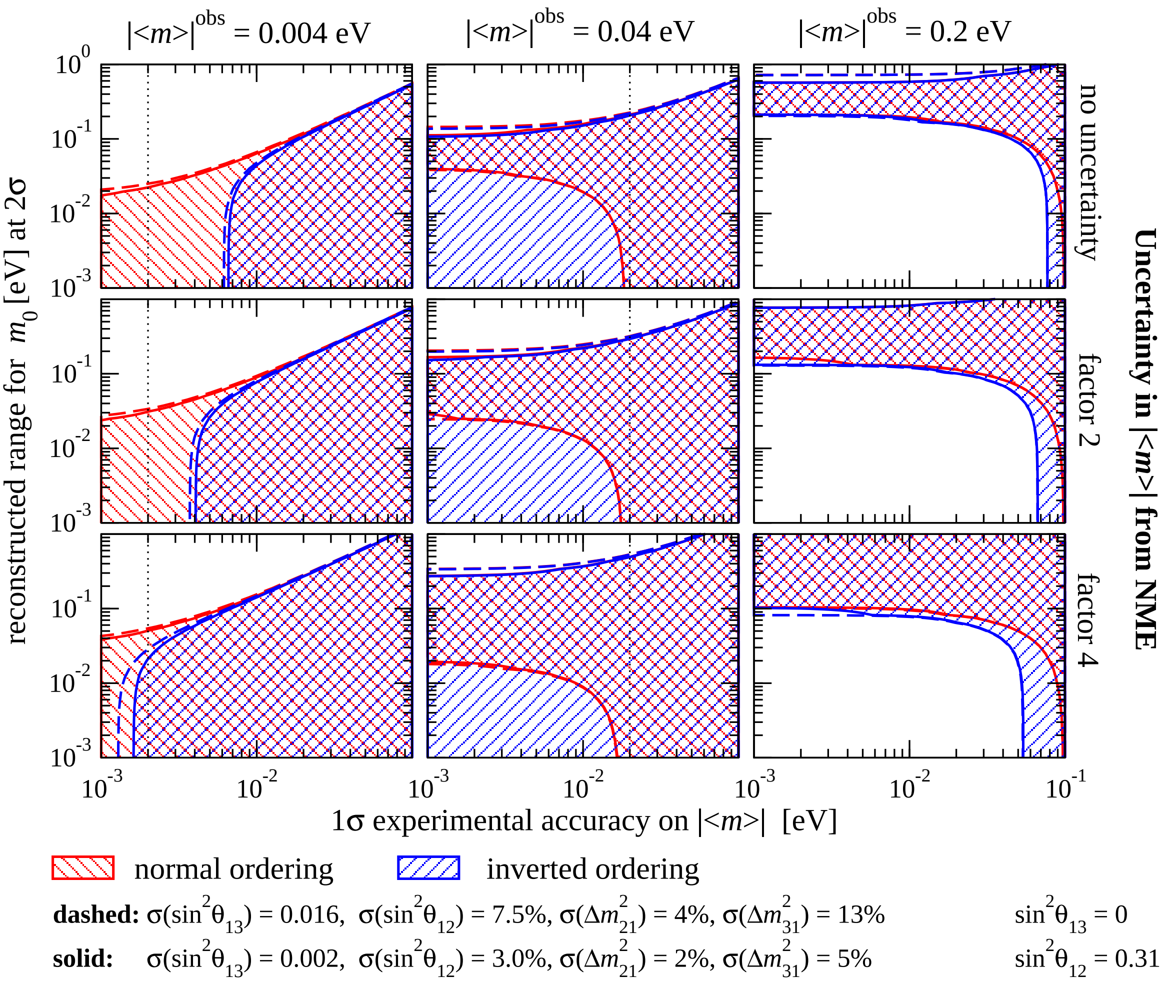

Figure 2: The reconstructed range for the lightest neutrino mass m_0 at 2σ C.L. for normal ($\Delta m^2_{31} > 0$) and inverted ($\Delta m^2_{31} < 0$) mass ordering as a function of the 1σ experimental error on $|{<}\,m\,{>}|^{\rm obs}$. The results are shown for three representative values $|{<}\,m\,{>}|^{\rm obs} = 0.004, 0.04, 0.2$ eV (columns of panels), and for fixed NME (first row), and an uncertainty of a factor of $F = 2$ and $F = 4$ in the NME (second and third rows). The figure is obtained using the current best fit values of Δm^2_{21}, $|\Delta m^2_{31}|$ and $\sin^2\theta_{12}$ (eqs. (8) and (10)), and $\sin^2\theta_{13} = 0$. The dashed (solid) lines correspond to the present (prospective) uncertainties on the oscillation parameters. To the left of the dotted lines, a positive signal is obtained at 2σ, whereas to the right only an upper bound can be stated.

between the NH, IH and QD spectra. As we have commented earlier, getting information on the possible hierarchical structure of the neutrino mass spectrum and on $\mathrm{sgn}(\Delta m^2_{31})$ are different, although not totally unrelated, problems. In this subsection we elaborate further on the issue, since a determination of the neutrino mass spectrum is fundamental for our understanding of neutrino mixing. We investigate what conclusions can be drawn at the 2σ C.L. on the neutrino mass spectrum from a result of a $(\beta\beta)_{0\nu}$-decay experiment, characterised by the observed value of $|{<}\,m\,{>}|$ and its experimental error. For given values of $|{<}\,m\,{>}|^{\rm obs}$, $\sigma_{\beta\beta}$, the uncertainty F in the NME and a fixed neutrino mass ordering, we

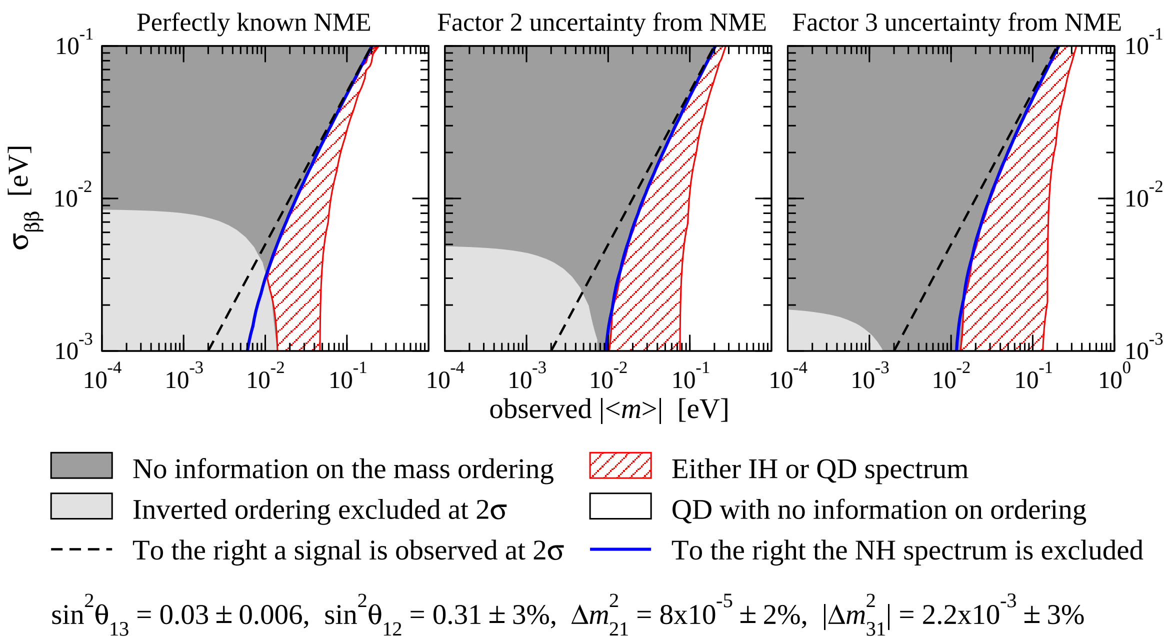

Figure 3: Information on the type of neutrino mass spectrum, inferred from data on $(\beta\beta)_{0\nu}$-decay as a function of the observed $|<m>|$ and its experimental error for three different assumptions on the NME uncertainty factor F (see text for details).

minimize the χ^2-function of eq. (28) with respect to m_0, α_{21} and α_{31}. If the χ^2-minimum is smaller than 4, we conclude that this type of ordering is allowed. In addition we test if the "data" are consistent with negligible m_0, which implies a hierarchical spectrum (more precisely, we test whether $\chi^2(m_0 = 0) \leq 4$). The results of our analysis are shown graphically in Fig. 3. For values of $|<m>|^{\text{obs}}$ and $\sigma_{\beta\beta}$ forming the dark shaded and white areas in the three panels, no information on $\text{sgn}(\Delta m^2_{31})$ can be obtained. The light shaded regions correspond to the case where $\text{sgn}(\Delta m^2_{31}) < 0$ (inverted mass ordering) can be excluded. In agreement with the results presented in the previous subsection, we find that this is only possible for $|<m>|^{\text{obs}} < 0.01$ eV and an experimental error well below 0.01 eV. To the right of the solid curve, the spectrum cannot be hierarchical for $\text{sgn}(\Delta m^2_{31}) > 0$, i.e. the possibility $m_1 \ll m_2 \ll m_3$ is ruled out (at 2σ). In the hatched region in this domain the "data" are still consistent with $m_0 = 0$ for the inverted ordering, i.e. with an IH spectrum. Hence, if a result within the hatched region is obtained, we can conclude that either $\text{sgn}(\Delta m^2_{31}) > 0$ and the spectrum is with partial hierarchy or of the QD type, or $\text{sgn}(\Delta m^2_{31}) < 0$ and the spectrum is IH ($m_0 \leq 0.02$ eV), or QD ($m_0 \gtrsim 0.1$ eV), or with partial hierarchy ($m_0^2 \sim |\Delta m^2_{\text{A}}|$). This situation corresponds, e.g. to the panels of the middle column in Fig. 2, or to the case when the lower branch at $m_0 \lesssim 0.01$ eV in the case of normal ordering (see Fig. 1) can be excluded. Finally, for sufficiently large values of $|<m>|^{\text{obs}}$, corresponding to the white regions in Fig. 3, the spectrum is of QD type and no information on $\text{sgn}(\Delta m^2_{31})$ can be obtained.

Let us add that these results are stable with respect to variations of the oscillation parameters within the present allowed ranges. In particular, they practically do not depend on the value of $\sin^2\theta_{13}$: no significant changes appear if, instead of the rather large value $\sin^2\theta_{13} = 0.03$ adopted in Fig. 3, smaller values are used. As discussed in Section 2, cosmol-

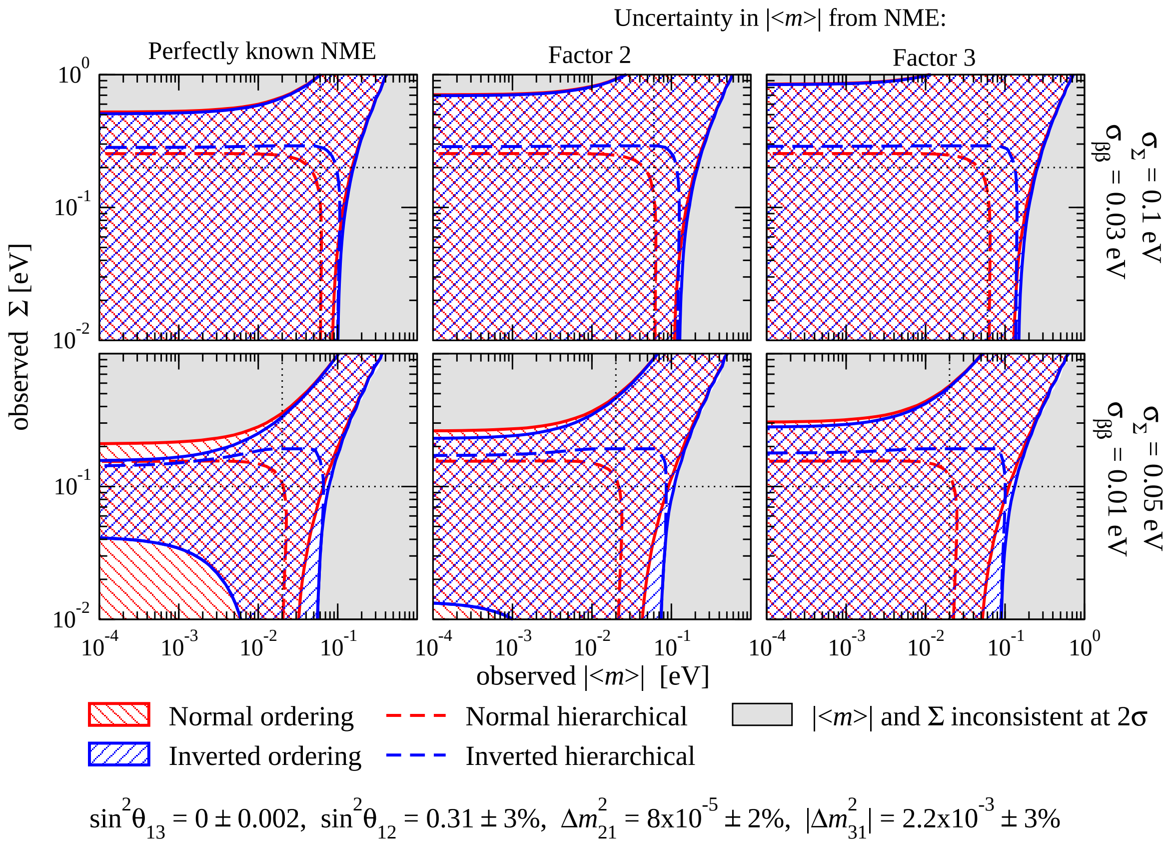

Figure 4: Information on the neutrino mass spectrum from a combination of $(\beta\beta)_{0\nu}$-decay data on $|{<}m{>}|$ and cosmological data on Σ. These results are obtained for different assumptions about the errors in the determination of $|{<}m{>}|$ and Σ (rows of panels) and the NME uncertainty factor F (columns of panels). We test whether the $(\beta\beta)_{0\nu}$-decay and cosmological data are consistent with each other, with normal or inverted mass ordering, with normal ordering and $m_0 = 0$ (NH ν-mass spectrum), or with inverted ordering and $m_0 = 0$ (IH spectrum), at 2σ C.L. The regions to the right of (above) the vertical (horizontal) dotted lines correspond to non-zero observed $|{<}m{>}|^{\rm obs}$ ($\Sigma^{\rm obs}$) at 2σ.

ogy provides a sensitive tool to constrain the sum of the neutrino masses $\Sigma \equiv m_1 + m_2 + m_3$. In the following we investigate what can be learned from data on $(\beta\beta)_{0\nu}$-decay, combined with information on Σ from cosmology. We include the latter by assuming an "observed" value of the sum of the neutrino masses $\Sigma^{\rm obs}$ with an experimental accuracy σ_Σ. Obviously, if $\Sigma^{\rm obs} > n\sigma_\Sigma$, cosmological observations would provide positive evidence for a non-zero $\Sigma^{\rm obs}$ at the $n\sigma$ C.L.; otherwise an upper bound is obtained. In Fig. 4 we show the results of a combined analysis of a $(\beta\beta)_{0\nu}$-decay result with information from cosmology as a function of the observed values of $|{<}m{>}|$ and Σ for two sets of representative experimental errors (upper and lower rows) and different assumptions on the uncertainty from the NME. For given values of $|{<}m{>}|^{\rm obs}$ and $\Sigma^{\rm obs}$ we look for the χ^2-minimum for each of the two possible mass orderings. If a minimum is less than 4, we conclude that the corresponding mass ordering is consistent with the data at 2σ. These regions are indicated by the hatched areas in Fig. 4. If the χ^2-minimum is bigger than 4 for both types of mass orderings, the corresponding values of $|{<}m{>}|^{\rm obs}$ and $\Sigma^{\rm obs}$ are not consistent at 2σ within the assumed uncertainties. Such a situation (shown as the shaded regions in Fig. 4) could either result from systematical effects not taken into account in the cosmological data, or can indicate that some mechanism beyond the light Majorana neutrino exchange is operating in $(\beta\beta)_{0\nu}$-decay [81]. We also test whether the data are consistent with *hierarchical* spectra, i.e. for each sign of Δm^2_{31} we test whether $\chi^2(m_0 = 0) \leq 4$. These regions are below the dashed lines in Fig. 4, within the corresponding hatched area.[13]

For experimental errors corresponding to $\sigma_{\beta\beta} = 0.03$ eV and $\sigma_\Sigma = 0.1$ eV adopted in the upper row of plots in Fig. 4, no distinction between $\text{sgn}(\Delta m^2_{31}) > 0$ and $\text{sgn}(\Delta m^2_{31}) < 0$ (i.e. normal and inverted ordering) is possible. However, some information can be obtained on whether the spectrum is *hierarchical* for a given $\text{sgn}(\Delta m^2_{31})$. In particular, the data from cosmology increase the ability to distinguish between IH and QD spectra in the case of $\text{sgn}(\Delta m^2_{31}) < 0$ if $|{<}m{>}| \simeq 0.1$ eV is observed. This situation corresponds to the case indicated by the hatched region in Fig. 3, where $(\beta\beta)_{0\nu}$-decay alone can only rule out the *normal hierarchical* spectrum.

For the more demanding experimental precision of $\sigma_{\beta\beta} = 0.01$ eV and $\sigma_\Sigma = 0.05$ eV, used in the lower row of plots in Fig. 4, a new possibility to distinguish between normal and inverted mass ordering appears. If the $(\beta\beta)_{0\nu}$-decay data give, e.g. a value of $|{<}m{>}|$ in the interval (0.04 - 0.07) eV for NME uncertainty factor $F = 2$, and the cosmological observations yield an upper bound $\Sigma^{\rm obs} \leq 2\sigma_\Sigma = 0.1$ eV, the IH spectrum can be established at 2σ C.L. This is a qualitatively new method to determine the mass ordering, emerging from a synergy between data from $(\beta\beta)_{0\nu}$-decay experiments and cosmology. Using $(\beta\beta)_{0\nu}$-decay data alone it is possible, in principle, to rule out the inverted ordering if $|{<}m{>}|^{\rm obs} < 0.01$ eV and the error $\sigma_{\beta\beta}$ is sufficiently small. However, as we have shown, the required error is exceedingly small: $\sigma_{\beta\beta} \lesssim$ few $\times\, 10^{-3}$ eV. In contrast, the conclusion following from a combination of $(\beta\beta)_{0\nu}$-decay and cosmological data is based on i) the observation of a value of $|{<}m{>}|$ compatible with those predicted for the IH spectrum, and ii) an upper bound on m_0 from cosmological data such that the region where the predictions for $|{<}m{>}|$ in the cases of normal and inverted hierarchies merge, can be excluded (see Fig. 1). Note that this possibility remains even if a factor 3 uncertainty in the NME is taken into account.

4.5 Constraining the Majorana CPV Phases

The possibility of establishing CP-violation due to Majorana phases through the observation of $(\beta\beta)_{0\nu}$-decay has been studied previously in [39, 41, 58]. In the following we re-consider this problem by applying the χ^2-method described in Section 4.2. We discuss first the case of QD spectrum and later consider the possibility of spectrum with inverted hierarchy, $\text{sgn}(\Delta m^2_{\rm A}) < 0$, and $|{<}m{>}|^{\rm obs}$ lying in the IH range of $(0.015 - 0.05)$ eV. As is clear from eqs. (15) and (20) and the discussion in Section 3, the dependence of $|{<}m{>}|$ on the phase α_{31} is suppressed by the small value of $\sin^2\theta_{13}$. Therefore, we concentrate on the determination of the phase α_{21}, while the dependence on α_{31} is taken into account implic-

[13]We adopt the convention to determine the compatibility of the $(\beta\beta)_{0\nu}$-measurement and cosmological data by evaluating the χ^2 for 1 degree-of-freedom. A motivation for this convention is provided by the so-called parameter-goodness-of-fit method discussed in [82]. In the context of that method the single degree-of-freedom corresponds to the one parameter, m_0, which is comon to $|{<}m{>}|$ and Σ.

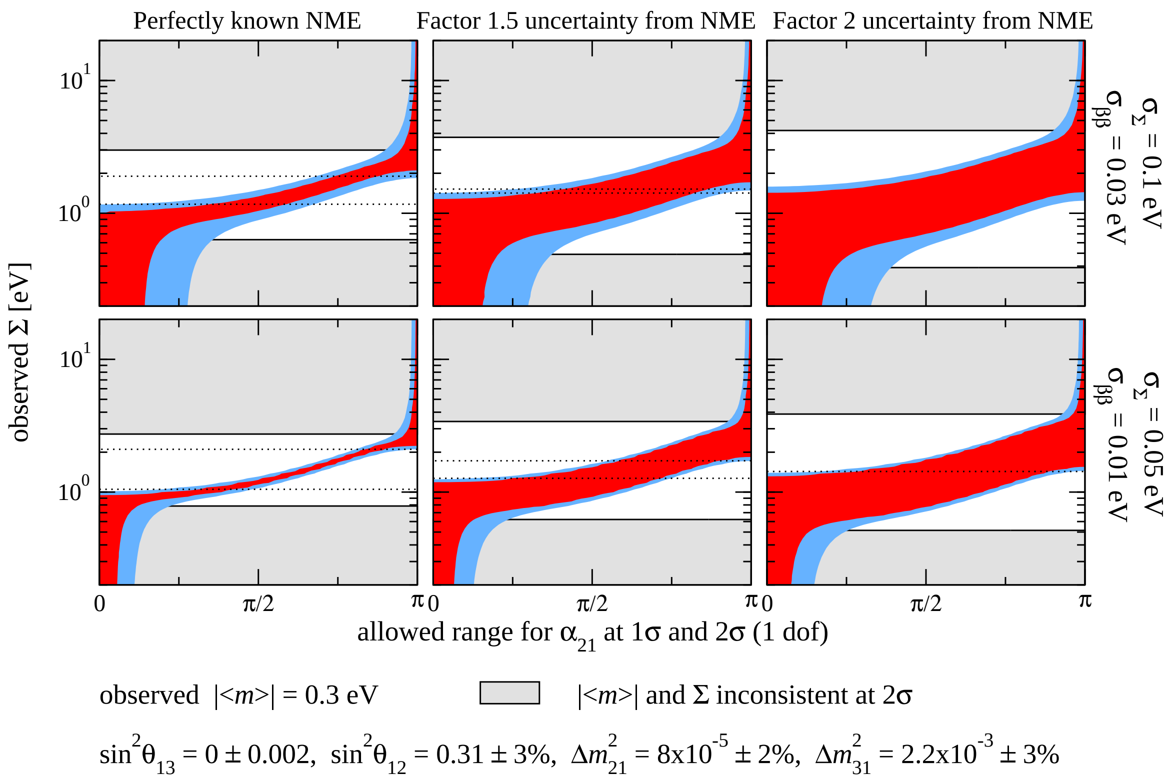

Figure 5: Allowed range for the Majorana phase α_{21} at 1 σ C.L. (dark-gray/red regions) and 2 σ C.L. (medium-gray/blue regions) for $|\!<\!m\!>\!| = 0.3$ eV as a function of the observed value of Σ. The shown results are obtained for two sets of assumed errors in the observed $|\!<\!m\!>\!|$ and Σ (rows of panels) and three values of the NME uncertainty factor F (columns of panels). For values of the parameters in the regions between the dotted lines, Majorana CP-violation can be established at 2σ.

itly by minimising the χ^2 with respect to it. For the oscillation parameters we will assume throughout this subsection uncertainties at the few percent level (precise numbers are given in the figures). Such a precision can be reached in upcoming oscillation experiments. In Fig. 5 we show the allowed range for the Majorana phase α_{21} for $|\!<\!m\!>\!|^{\rm obs} = 0.3$ eV as a function of the observed mean value of Σ. The 1σ (2σ) range is obtained by the condition $\Delta\chi^2(\alpha_{21}) = \chi^2(\alpha_{21}) - \chi^2_{\rm min} \leq 1\,(4)$. Since the allowed range is determined by $\Delta\chi^2$ with respect to the χ^2-minimum, there is always an "allowed region", irrespectively of whether $\Sigma^{\rm obs}$ is consistent with the adopted value of $|\!<\!m\!>\!|^{\rm obs}$. We indicate in Fig. 5 the region where the "results" of $(\beta\beta)_{0\nu}$-decay experiment and cosmological observations are inconsistent ($\chi^2_{\rm min} \geq 4$) by the light shading. Majorana CP-violation can be established if both $\alpha_{21} = 0$ and $\alpha_{21} = \pi$ can be excluded. The relevant regions are indicated by the horizontal dotted lines in Fig. 5. One observes that for $\sigma_{\beta\beta} = 0.03$ eV and $\sigma_\Sigma = 0.1$ eV used in the upper row of plots, already an uncertainty of a factor of 1.5 in the NME makes it practically impossible to establish CPV. Our results show, in agreement with the results of the previous studies [41, 58], that establishing Majorana CP-violation due to α_{21} is very challenging: the errors in the observed $|\!<\!m\!>\!|$ and Σ should not exceed approximately 10% and the NME has to be known within a factor $F \lesssim 1.5$. Although establishing Majorana

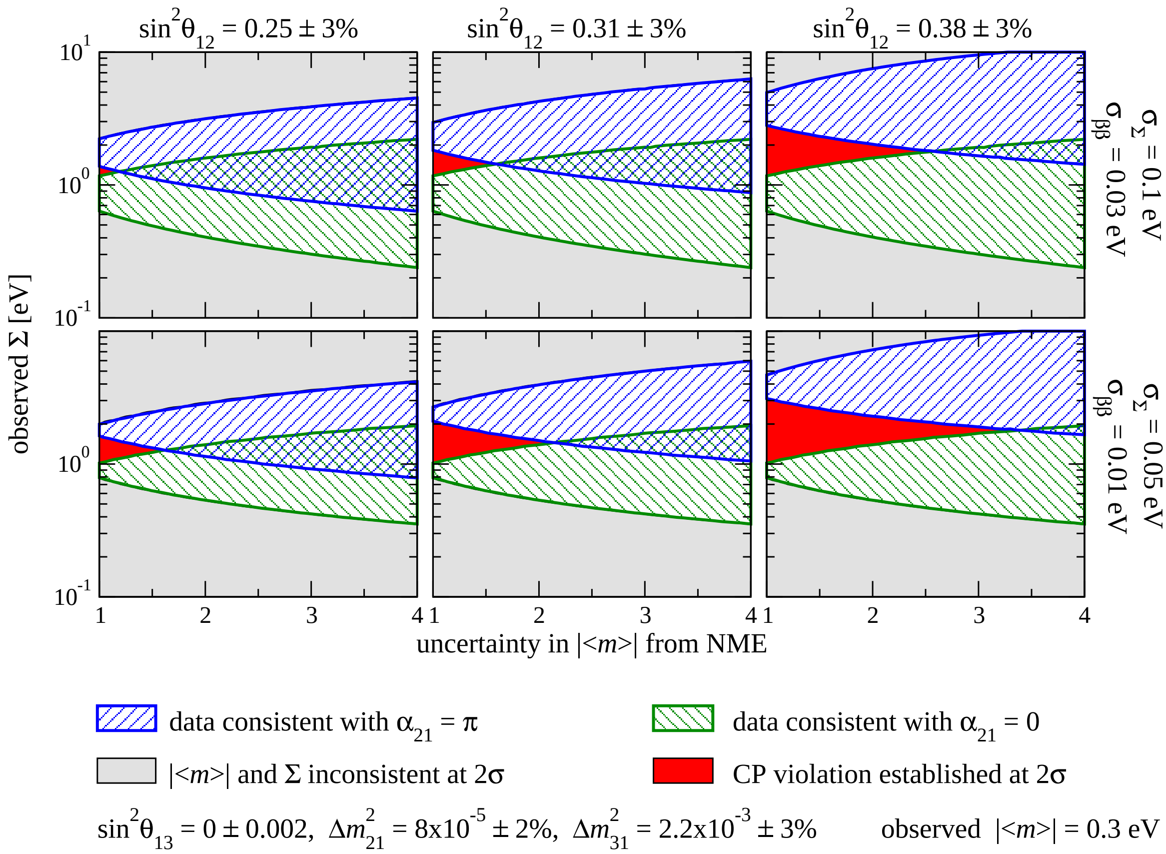

Figure 6: Constraints on the Majorana phase α_{12} at 95% C.L. from an observed $|<m>|^{\rm obs} = 0.3$ eV and (cosmological) "data" on Σ, as a function of the NME uncertainty factor F. Shown are the regions in which i) the "data" are consistent with one of the CP-conserving values $\alpha_{12} = 0$ or π (hatched), ii) $\Sigma^{\rm obs}$ is inconsistent with $|<m>|^{\rm obs} = 0.3$ eV (light-shaded), and iii) Majorana CP-violation is established (red/dark-shaded). The results are presented for three values of $\sin^2\theta_\odot$ within the currently allowed range (columns of panels), and for two choices of the experimental accuracies for $|<m>|$ and Σ (rows of panels).

CPV would be a very difficult task, it could be possible to exclude a certain fraction of the full parameter space of the phase α_{21} [14] by using the data on $|<m>|$ and Σ. In particular, in many cases it could be possible to exclude one of the CP-conserving values of α_{21}, $\alpha_{21} = 0$ or $\alpha_{21} = \pi$, corresponding to specific relative CP-parities of the neutrinos ν_1 and ν_2. The sensitivity to α_{21} depends significantly on the value of the mixing angle $\theta_\odot$ [41, 58]. As is discussed in Sec. 3 (see eq. (22)), for fixed m_0 the allowed range of $|<m>|$ is given by $m_0 \cos 2\theta_\odot \leq |<m>| \leq m_0$. Therefore, the allowed range increases for smaller values of $\cos 2\theta_\odot$, which makes it easier to exclude the extreme values of $|<m>|$, corresponding to the CP-conserving configurations. This effect is clearly shown in Fig. 6, where the three columns of panels correspond to different values of $\sin^2\theta_\odot$. We use the current best fit point as well as values between the present 2σ and 3σ limits. In Fig. 6 we assume an observation

[14] This situation is similar to the case of the determination of the Dirac CP phase δ by long-baseline oscillation experiments, see [83] for a recent discussion and a list of references.

of $|<\!m\!>|^{\rm obs} = 0.3$ eV, which implies a QD spectrum. Adopting representative values for the experimental accuracies on $|<\!m\!>|$ and Σ, and scanning values of $\Sigma^{\rm obs}$ and the uncertainty in the NME, we test first whether $\Sigma^{\rm obs}$ and $|<\!m\!>|^{\rm obs}$ are consistent at 2σ C.L. ($\chi^2_{\rm min} \leq 4$). If they are, we test whether the phase α_{21} is consistent with the CP-conserving values 0 and π ($\chi^2(\alpha_{21} = 0, \pi) \leq 4$). These two cases are marked by the hatched regions in Fig. 6. The double-hatched areas, where the regions for $\alpha_{21} = 0$ and π overlap, correspond to the worst situation, where no information on α_{21} can be obtained and the full range $[0, \pi]$ is allowed by the data. If the values of $\Sigma^{\rm obs}$ and $|<\!m\!>|^{\rm obs}$ are consistent and both CP-conserving solutions for α_{21} can be excluded, Majorana CP-violation can be established at 95% C.L., and we indicate the corresponding regions in red/dark-shading. Clearly, establishing Majorana CP-violation becomes possible only under rather specific conditions. For $\sin^2\theta_\odot \gtrsim 0.31$ and $\sim 10\%$ errors in the measured $|<\!m\!>|^{\rm obs}$ and $\Sigma^{\rm obs}$ (upper middle and right panels in Fig. 6), the NME has to been known to better than within a factor of 1.5. For smaller values of the errors, $\sigma_{\beta\beta} \sim 0.01$ eV and $\sigma_\Sigma \sim 0.05$ eV, Majorana CP-violation could be established even for $F \cong 2$ (lower middle and right panels). If, however, $\sin^2\theta_\odot \cong 0.25$, the NME uncertainty has to be small, $F \leq 1.5$ and the indicated high precision in the measurement of $|<\!m\!>|^{\rm obs}$ and $\Sigma^{\rm obs}$ has to be achieved. Finally, the Majorana phase α_{21} has to have a value approximately in the interval $\sim (\pi/4 - 3\pi/4)$.

Consider next the possibility to establish Majorana CP-violation assuming that the ν mass spectrum is known to be with inverted ordering, $sgn(\Delta m^2_{\rm A}) < 0$, and that the observed value of $|<\!m\!>|$ lies in the IH region of a few $\times 10^{-2}$ eV. Knowing that this would require a rather precise measurement of $|<\!m\!>|$, we use for the experimental error on $|<\!m\!>|$ the value $\sigma_{\beta\beta} = 4 \times 10^{-3}$ eV. For the sum of neutrino masses Σ we adopt the error $\sigma_\Sigma = 4 \times 10^{-2}$ eV. In Fig. 7 we show the sensitivity to Majorana CP-violation for three representative mean values of $|<\!m\!>|$ from the IH region, $|<\!m\!>|^{\rm obs} = 0.018;\ 0.032;\ 0.047$ eV, and two mean values of $\sin^2\theta_\odot$, $\sin^2\theta_\odot = 0.31;\ 0.38$. The allowed regions in all panels of this figure are bounded from below by a straight line at $\Sigma^{\rm obs} = 0.014$ eV: below this value Σ becomes inconsistent with the adopted value of $|\Delta m^2_{\rm A}|$.

The upper row of panels corresponds to the present best fit point of $\sin^2\theta_\odot$. For $|<\!m\!>|^{\rm obs} = 0.018$ eV (left panel), the 2σ interval of allowed values of $|<\!m\!>|$ *always* includes the minimal value of $|<\!m\!>|$ for the IH spectrum (see eq. (18)): $|<\!m\!>|^{\rm min} \cong \sqrt{|\Delta m^2_A|}\cos 2\theta_\odot$. The latter corresponds to the CP-conserving value $\alpha_{21} = \pi$. Thus, for the chosen values of $|<\!m\!>|^{\rm obs}$, $\sigma_{\beta\beta}$, σ_Σ and $\sin^2\theta_\odot$, it is impossible to establish Majorana CP-violation. However, if $F \leq 3$, it will be possible to conclude that α_{21} has a nonzero value, $\alpha_{21} \neq 0$. Thus, if CP is conserved, the neutrinos ν_1 and ν_2 cannot have the same CP-parities.

The value $|<\!m\!>|^{\rm obs} = 0.032$ eV adopted in the middle panel corresponds to the case when $|<\!m\!>|$ (with the experimental uncertainty included) satisfies $|<\!m\!>|^{\rm min} < |<\!m\!>| < |<\!m\!>|^{\rm max}$, $|<\!m\!>|^{\rm max}$ being maximal value of $|<\!m\!>|$ predicted in the case of IH spectrum (see eq. (18)), $|<\!m\!>|^{\rm max} \cong \sqrt{|\Delta m^2_A|}$. If an upper bound on m_0 is provided by a constraint on Σ, Majorana CP-violation can be established, as evident from the red/dark-shaded region in the panel. If the observed value of Σ becomes too large, the "data" becomes consistent with the "upturn" of the IH-branch (see Fig. 1), which implies that the CP-conserving value $\alpha_{21} = \pi$ is allowed. For $\Sigma^{\rm obs} \lesssim 0.2$ eV and uncertainties in the NME $F \gtrsim 1.5 - 2$, the "data" become consistent with $|<\!m\!>|^{\rm max}$, i.e. with $\alpha_{21} = 0$.

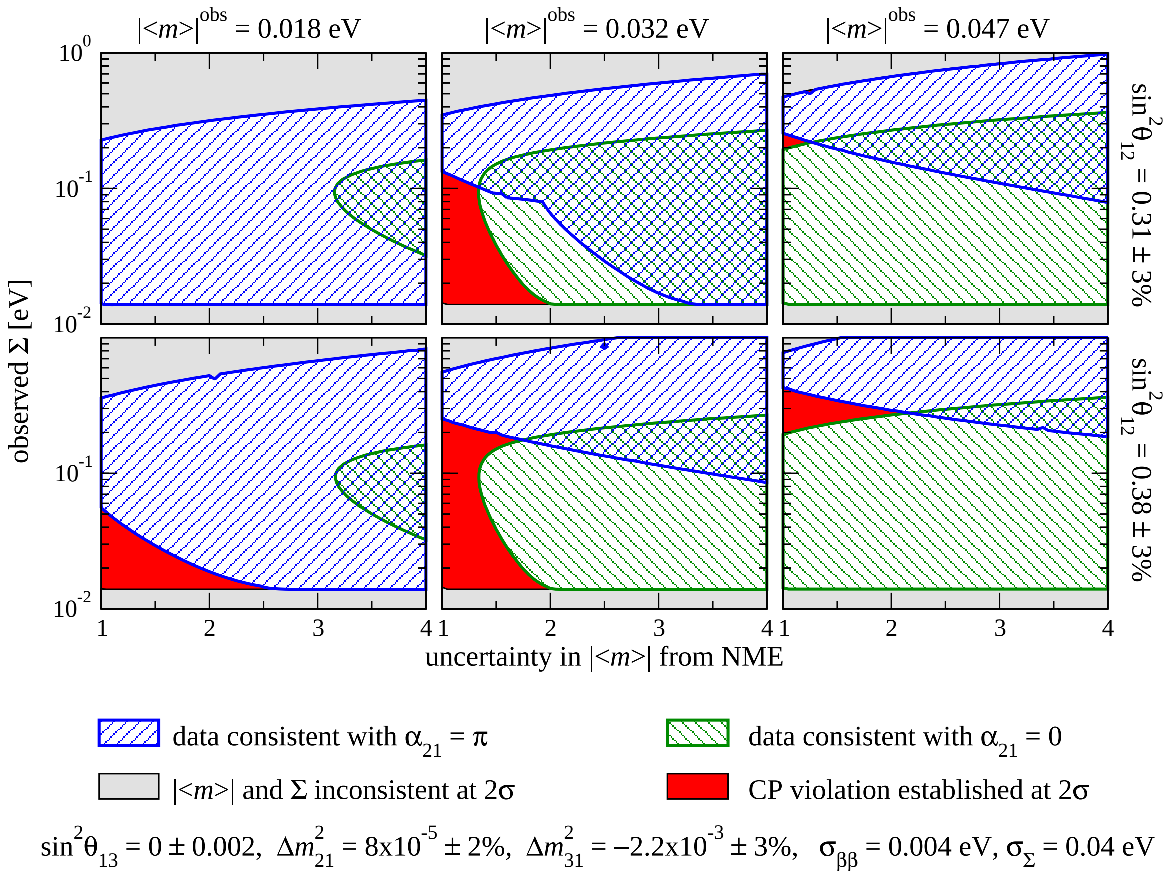

Figure 7: Constraints on the Majorana phase α_{12} at 95% C.L. for the inverted mass ordering from observed values $|{<}m{>}|^{\rm obs} = 0.018, 0.032, 0.047$ eV and (cosmological) "data" on Σ, as a function of the NME uncertainty factor F. Shown are the regions in which i) the "data" are consistent with one of the CP-conserving values $\alpha_{12} = 0$ or π (hatched), ii) $\Sigma^{\rm obs}$ is inconsistent with $|{<}m{>}|^{\rm obs}$ (light-shaded), and iii) Majorana CP-violation is established (red/dark-shaded). The upper (lower) row of panels corresponds to $\sin^2\theta_\odot = 0.31$; (0.38).

For the third representative value of $|{<}m{>}|^{\rm obs} = 0.047$ eV (right panels), $|{<}m{>}|^{\rm max}$ lies in the 2σ interval of allowed values of $|{<}m{>}|$. This implies that Majorana CP-violation can only be established if the "data" on Σ constrains m_0 precisely in the range corresponding to a neutrino mass spectrum with *partial inverted hierarchy*, such that neither $|{<}m{>}|^{\rm max}$ (i.e. the horizontal branch at $\alpha_{21} = 0$), nor the upturn of the lower (minimal) branch at $\alpha_{21} = \pi$, are compatible with the "data". As can be seen in Fig. 7, this is marginally possible for $\sin^2\theta_\odot = 0.31$ (upper row), but some window exists for $\sin^2\theta_\odot = 0.38$ (lower row) if the NME uncertainty factor $F \leq 2$.

5 Conclusions

In the present article we have reanalysed the potential contribution of future $(\beta\beta)_{0\nu}$-decay experiments to the studies of neutrino mixing. We have considered 3-ν mixing and assumed massive Majorana neutrinos and $(\beta\beta)_{0\nu}$-decay generated only by the $(V-A)$ charged current weak interaction via the exchange of the three Majorana neutrinos. In this framework we investigated which information can be obtained from a measurement of the effective Majorana mass $|<m>|$ i) on the type of neutrino mass spectrum (NH, IH, QD, etc.) ii) on the absolute scale of neutrino masses, and iii) on the Majorana CP-violating phases. As input in the analysis we used the results of recent studies of the prospective precision that can be achieved in the future measurements of neutrino oscillation parameters on which $|<m>|$ depends. We performed a χ^2 analysis taking into account experimental and theoretical errors, as well as the uncertainty implied by the imprecise knowledge of the corresponding nuclear matrix element (NME).

We show how the possibility to discriminate between the NH, IH and QD spectra depends on the mean value and the experimental error of $|<m>|$, and on the NME uncertainty. Furthermore, we combine the information on $|<m>|$ from a $(\beta\beta)_{0\nu}$-decay experiment, with a constraint on the sum of the neutrino masses, Σ, which can be obtained from cosmological observations. In this case, we investigate the role of the accuracies on $|<m>|$ and Σ, as well as on the NME uncertainty, in determining the type of neutrino mass spectrum. The constraints on Majorana CP-violation phases in the neutrino mixing matrix, that can be obtained from a measurement of $|<m>|$ and Σ in the cases when i) the observed $|<m>| \sim$ few $\times\, 10^{-1}$ eV (QD spectrum), and ii) $\mathrm{sgn}(\Delta m^2_{\mathrm{A}}) < 0$ and the observed $|<m>| \sim$ few $\times\, 10^{-2}$ eV, are also analyzed in detail. We have estimated the required experimental accuracies on both types of measurements, and the required precision in the NME permitting to address the issue of Majorana CP-violation in the lepton sector.

Our results show that, in general, getting quantitative information on the neutrino mass and mixing parameters from a measurement of the $(\beta\beta)_{0\nu}$-decay half-life is rather insensitive to the errors on the input neutrino oscillation parameters as long as the errors are smaller than $\sim 10\%$. However, constraints on the absolute neutrino mass scale, on the type of neutrino mass spectrum and on the Majorana CP-violation phase one can obtain depend critically on the measured mean value of $|<m>|$ (and Σ), on the precision reached in the measurement of $|<m>|$ (and Σ), and on the uncertainty in the knowledge of the value of the relevant $(\beta\beta)_{0\nu}$-decay nuclear matrix element. The most challenging of these physics goals is obtaining quantitative information on Majorana CP-violation phases. The sensitivity to the latter depends crucially *also* on the value of $\sin^2\theta_\odot$. Establishing Majorana CP-violation using data on $|<m>|$ and Σ in the case of QD spectrum, for instance, would require for $\sin^2\theta_\odot \cong 0.31$, a $\sim 10\%$ (or smaller) errors in the measured $|<m>|^{\mathrm{obs}}$ and Σ^{obs} and knowledge of the relevant NME with an uncertainty corresponding to a factor $F \leq 1.5$. For smaller values of the errors and/or larger values of $\sin^2\theta_\odot$, say $\sin^2\theta_\odot \cong 0.38$, it could be possible to obtain evidence of Majorana CP-violation at 2σ C.L. even for $F \cong 2$. If, however, $\sin^2\theta_\odot \cong 0.25$, exceedingly high precision in the measurements of $|<m>|$ and Σ and NME uncertainty smaller than 1.5 is required. In all these cases the Majorana phase α_{21} has to have also a value approximately in the interval $\sim (\pi/4 - 3\pi/4)$.

Future $(\beta\beta)_{0\nu}$-decay experiments have a remarkable physics potential. They can establish the Majorana nature of neutrinos with definite mass ν_j. If the latter are Majorana particles, the $(\beta\beta)_{0\nu}$-decay experiments can provide constraints on the absolute scale of neutrino masses and on the type of neutrino mass spectrum. They can also provide unique information on the Majorana CP-violation phases present in the PMNS neutrino mixing matrix. The measurement of $|<m>|$ (and Σ) with sufficiently small error and sufficiently precise knowledge of the values of the relevant $(\beta\beta)_{0\nu}$-decay nuclear matrix elements ($F < 2$) is crucial for obtaining significant quantitative information on the neutrino mass and mixing parameters from a measurement of $(\beta\beta)_{0\nu}$-decay half-life. The remarkably challenging physics goal of getting evidence for Majorana CP-violation in the lepton sector could possibly be achieved only if *in addition* $\sin^2\theta_\odot$ and the Majorana CP-violation phase α_{21} have favorable values.

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