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The α' stretched horizon in Heterotic string

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Abstract: The linear α' corrections and the field redefinition ambiguities are studied for half-BPS singular backgrounds representing a wrapped fundamental string. It is shown that there exist schemes in which the inclusion of all the linear α' corrections converts these singular solutions to black holes with regular horizon for which the modified Hawking-Bekenstein entropy is in agreement with the statistical entropy.

Keywords: String theory, alpha-prime corrections, horizon, black hole.

Contents

1. Introduction

The massless field of helicity two in the spectrum of string theory is identified as the gravitational field since its low energy effective action around flat space-time coincides with the Einstein-Hilbert action. This identification sets the subleading string corrections as the quantum corrections to gravity and allows one to ask if and how quantum corrections preserve or change the properties of classical backgrounds. In particular one may ask if the subleading string corrections induce a regular horizon on the singular classical geometries which have an entropy associated to them.

Amongst these singular classical geometries are the half BPS null singular ones which represent a wrapped fundamental string with general momentum and winding numbers [1]. These null singular geometries have a statistical entropy associated to them since string states with given momentum and winding numbers are degenerate [2]. It is conjectured that quantum effects convert these singular geometries to black holes with regular horizon whose thermodynamical entropy is in agreement with the statistical one.

It is known that the the leading world-sheet corrections of Heterotic string includes the square of the Riemann tensor. Ref [3], motivated by [4], observed that the inclusion of the square of the Riemann tensor and its supersymmetric partners in $D = 4$ [5, 6, 7, 8, 9, 10, 11, 12, 13, 14] induces a local horizon with geometry $AdS_2 \times S^2$ on these backgrounds and for which the modified Hawking-Bekenstein entropy [15, 16, 17] is in agreement with the statistical one. This observation renewed interest in the subject [18, 19, 20, 21, 22, 23, 24, 25]. Ref. [21, 22, 26] introduced an entropy formalism and concluded that the inclusion of the Gauss-Bonnet action as a part of the linear α' corrections in an arbitrary dimension induces a local horizon with geometry $AdS_2 \times S^{D-2}$ whose modified Hawking-Bekenstein entropy is in agreement with the statistical one up to a numerical constant factor.

In this note we present a way to calculate all the linear α' corrections in an arbitrary dimension and we study how they may change these null singular backgrounds to black holes. The note is organised in the following way;

In the second section we review the classical solutions representing a wrapped fundamental string on a two cycle. We realise them as ten dimensional backgrounds composed of metric, NS two form and dilaton first compactified on a torus of appropriate dimensionality to $D+1$ dimensional space-time and then through KK compactification on a circle to a D dimensional space-time.

In the third section we review how the α' corrections can be computed. We present the linear α' corrections in Heterotic theory to backgrounds of metric, NS two form and dilaton obtained from scattering amplitude considerations [27, 28]. We study the field redefinition ambiguities. We require that the generalisation of the Einstein tensor is covariantly divergence free. This requirement fixes the curvature squared terms to the Gauss-Bonnet Lagrangian keeping some of the field redefinition ambiguity parameters untouched.

In the fourth section we employ the compactification process of the first section to account for all the linear α' corrections in lower dimensions using the corrections in ten dimensions. We show that there exist schemes in which the inclusion of all the linear α' corrections in an arbitrary dimension gives rise to a local horizon with geometry $AdS_2 \times S^{D-2}$ whose modified Hawking-Bekenstein entropy is in agreement with the statistical entropy. We address the role of the existence of a smooth solution connecting the local horizon to asymptotic infinity and the effects of higher order α' corrections.

In the fifth section the conclusions are presented.

2. The tree-level singular background

The low energy effective action of the critical heterotic string theory for the metric ($\boldsymbol{g}$), the NS two-form ($\boldsymbol{B}$) and the dilaton ($\boldsymbol{\phi}$) reads

$$\boldsymbol{S}^{(10)} \sim \int d^{10}\boldsymbol{x}\; \boldsymbol{L}^{(10)} = \frac{1}{32}\int d^{10}\boldsymbol{x}\,\sqrt{-\boldsymbol{g}}\,e^{-2\boldsymbol{\phi}}\,(\boldsymbol{R}_{\text{Ricci}} + 4|\boldsymbol{\nabla}\boldsymbol{\phi}|^2 - \frac{1}{3}\boldsymbol{H}_{ijk}\boldsymbol{H}^{ijk})\,, \tag{2.1}$$

where

$$\boldsymbol{H}_{ijk} = \frac{3}{2}\boldsymbol{B}_{[ij,k]}\,. \tag{2.2}$$

The bold symbols will be used to represent the fields in ten dimensions. We are interested in the extrema of (2.1) whose fields configuration follows

$$\boldsymbol{ds^2} = \sum_{\mu,\nu=1}^{D} \boldsymbol{g}_{\mu\nu}(x)dx^\mu dx^\nu + 2\boldsymbol{g}_{y\mu}(x)\, dy\, dx^\mu + \boldsymbol{g}_{yy}(x)dy^2 + \sum_{m=D+1}^{10} dz_m^2\,, \tag{2.3}$$

$$\boldsymbol{B} = \boldsymbol{B}_{\mu\nu}(x)dx^\mu \wedge dx^\nu + \boldsymbol{B}_{y\mu}(x)dx^y \wedge dy\,, \tag{2.4}$$

$$\boldsymbol{\phi} = \boldsymbol{\phi}(x)\,, \tag{2.5}$$

$$y \sim y + 2\pi\,. \tag{2.6}$$

These extrema are examples of trivial compactification on a torus of appropriate dimensionality from "10" dimensions to a "$D+1$" dimensional space-time and then KK compactification on a circle to a D dimensional space-time. If one represents non-trivial components of the ten dimensional fields by

$$\begin{aligned} \boldsymbol{g}_{yy}(x) &= T^2\,, & \boldsymbol{g}_{y\mu}(x) &= 2A^{(1)}T^2\,, \\ \boldsymbol{g}_{\mu\nu}(x) &= g_{\mu\nu} + 4T^2 A_\mu^{(1)} A_\nu^{(1)}\,, & \boldsymbol{B}_{y\mu}(x) &= 2A_\mu^{(2)}\,, \\ 2\boldsymbol{\phi}(x) &= 2\phi + \ln T\,, & \boldsymbol{B}_{\mu\nu}(x) &= B_{\mu\nu} + 2(A_\mu^{(1)}A_\nu^{(2)} - A_\nu^{(1)}A_\mu^{(2)})\,, \end{aligned} \tag{2.7}$$

then the induced action for the new fields - $g, A^{(1)}, A^{(2)}, B, T$ and ϕ- reads

$$\begin{aligned} S &= \int d^D x L \\ &= \frac{1}{32}\int d^D x\sqrt{-g}e^{-2\phi}(R_{\text{Ricci}} + 4|\nabla\phi|^2 - \frac{|\nabla T|^2}{T^2} - \frac{|dB|^2}{12} - T^2|dA^{(1)}|^2 - \frac{|dA^{(2)}|^2}{T^2}), \end{aligned} \tag{2.8}$$

where R_{Ricci} is the Ricci scalar of $g_{\mu\nu}$, and an integration by parts is understood

$$\boldsymbol{L}^{(10)} - L \,=\, 2\sqrt{-g}\;\nabla^\mu(e^{-2\phi}\,\frac{\nabla_\mu T}{T})\,. \tag{2.9}$$

We refer to (2.8) as the induced action, and to x_μ and (z^μ,y) respectively as the large dimensions and as the compactified space. Due to the form of the induced action it is natural to interpret $A^{(1)}$ and $A^{(2)}$ as different $U(1)$ gauge connections in the large dimensions.[1] A family of the extrema of the compactified action is given by

$$ds^2_{string} = -\,e^{4\phi(r)}\,dt^2 \,+\, dr^2 \,+\, r^2\,d\Omega^2_{D-2}\,, \tag{2.10}$$

$$e^{-4\phi(r)} = \frac{(r^{D-3}+2W)\,(r^{D-3}+2N)}{r^{2(D-3)}}\,, \qquad T(r) \,=\, \sqrt{\frac{r^{D-3}+2N}{r^{D-3}+2W}}\,, \tag{2.11}$$

$$A_\tau^{(1)}(r) = -\,\frac{N}{r^{D-3}+2N}\,, \qquad A_\tau^{(2)}(r) \,=\, -\,\frac{W}{r^{D-3}+2W}\,, \tag{2.12}$$

[1] We are considering backgrounds of vanishing spin(32)/Z_2 or $E_8 \times E_8$ connections. In the presence of these connections there is an anomaly contribution to $d\boldsymbol{H}$, identified as a part of the α' corrections [29].

where N and W are two arbitrary numbers labelling the solution. We only consider the case where N and W are both positive. These backgrounds are constructed in [1] as singular limits of regular black-holes obtained by applying a solution generating transformation [30, 31] on a higher dimensional Kerr metric. Here we use the notation of [32]. Ref. [1] proved that they break half of the ten dimensional supersymmetries leaving eight unbroken supersymmetry parameters. These backgrounds are null-singular, i.e. the horizon coincides with the singularity. They represent BPS states of an elementary string carrying n units of momentum and w units of winding charges along S^1 of y coordinates where [32]

$$n = \frac{(D-3)\Omega_{D-2}}{4\pi} N \,, \tag{2.13}$$

$$w = \frac{(D-3)\Omega_{D-2}}{4\pi} W \,. \tag{2.14}$$

For general values of N and W a tachyon instability may exist around the singularity, reminiscent of the tachyon instability outside the horizon of Euclidean black holes presented in [33, 34]. We focus on the cases where $N \sim W$ and this instability is not present.

An entropy may be associated to these backgrounds since in general there exists more than one state of the Heterotic string carrying w units of winding and n units of momentum along S^1 of y coordinate. For large n and w the degeneracy of these states grows as $e^{4\pi\sqrt{nw}}$ [35]. Thus the entropy, defined by the logarithm of the degeneracy of the states, is given by:

$$S_{\text{statistical}} = 4\pi\sqrt{nw} \,, \tag{2.15}$$

when n and w are large. We refer to this entropy as the statistical entropy. A dilemma will arise as soon as the statistical entropy is associated to these tree-level backgrounds since they are singular and do not possess a regular event horizon to which the thermodynamical properties can be connected. This dilemma can be resolved in either of the following ways,

I. Statistical entropy should not be associated to these backgrounds.

II. Thermodynamical properties should be expressed in term of something else, in place of the event horizon, which null-singular geometries possess.

III. The subleading string corrections will induce an event horizon and the horizon cloaks the singularity.

Of the above possibilities, the first seems unnatural since the statistical entropy is associated to regular black holes [36, 37, 38] and these singular backgrounds are a limit of regular black holes. The fact that both the Euclidean path integral approach[2]

[2]Note that in string theory the presence of the tachyon-like winding modes of the tachyon wrapped around the Euclidean time which survive GSO projection [33, 34] adds to the known disturbing aspect [39] of the Euclidean approach.

[40] and the Noether current method [15, 16] express entropy of a given black hole in term of its event horizon is not sufficient to conclude that entropy could not be associated to geometries without the event horizon. We would like to point out that Mathur and Lunin's description of the entropy [41] may resolve the dilemma in the second way. In this note we focus on the third possibility. It is known that the thermodynamical entropy of the possible induced horizon on (2.10) is in agreement with the statistical entropy [1, 42] up to a numerical factor. Here we discuss the appearance of the stretched horizon itself.

3. The α' corrections

String theory provides two kind of perturbative corrections to a given background; the string loop corrections and the string world-sheet (α') corrections. The string coupling constant of (2.10), $g_s^2 = g_0^2 e^{2\phi}$, is

$$g_s^2 = g_0^2 \frac{r^{D-3}}{\sqrt{(r^{D-3}+2W)(r^{D-3}+2N)}} \leq g_0^2 \,, \tag{3.1}$$

where g_0 is an arbitrary parameter. We choose a sufficiently small value for g_0. Thus we ignore the string loop corrections. The α' corrections to the Lagrangian read

$$L \;=\; L^{(0)} \,+\, \alpha' \, L^{(1)} \,+\, \alpha'^2 \, L^{(2)} \,+\, \cdots \,, \tag{3.2}$$

where $L^{(0)}$ stands for the tree-level Lagrangian and the rest is its successive subleading corrections. This series may not make sense for (2.10) since each term of the α' series diverges at its singularity. However note that the α' corrections change the background itself

$$g \;\rightarrow\; g = g^{(0)} \,+\, \alpha' \, g^{(1)} \,+\, \alpha'^2 \, g^{(2)} \,+\, \cdots \,, \tag{3.3}$$

and the α'-corrected metric, possibly, can have a horizon outside which the α' expansion makes sense. Also the α' corrections to the string coupling constant may remain finite outside the horizon and the string loop corrections can be ignored consistently. The perturbative α' corrections can be computed in the following ways

- From the scattering amplitudes of string on sphere as done in [27, 28, 43]. This method gives the Low Energy Effective action up to a perturbative field redefinition since field redefinitions do not alter the scattering amplitudes.

- Requiring exact conformal symmetry in the corresponding sigma model as done in [44, 45, 46, 47, 48, 49, 48] . In this method a regularisation and a renormalisation scheme should be chosen prior to computing the beta functions. Different schemes are related to each other by a perturbative field redefinition.

- Calculating the LEE action in the Heterotic closed string field theory [50]. This computation has not yet been completed. However it does not fix the perturbative field redefinition ambiguity since there remains the freedom to redefine the fields [51].

The first two methods give the same action up to a perturbative field redefinition ambiguity as the result of the consistency of the string theory around flat space-time [52, 53]. The outcome of the last method should be in agreement with those of the former ones. The linear α' corrections in Heterotic theory derived from string amplitude considerations read [27, 28]

$$\begin{aligned} S^{(1)}_{MT} &= \int d^{10}x\sqrt{-\det g}\; e^{-2\boldsymbol{\phi}}\boldsymbol{L}^{(1)}_{MT}\,, \qquad (3.4)\\ \boldsymbol{L}^{(1)}_{MT} &= \frac{1}{4}\boldsymbol{R}_{klmn}\boldsymbol{R}^{klmn} + \boldsymbol{R}_{klmn}\boldsymbol{H}_p{}^{kl}\boldsymbol{H}^{pmn} + \\ &\quad -\frac{1}{2}\boldsymbol{H}_k{}^{mn}\boldsymbol{H}_{lmn}\boldsymbol{H}^{kpq}\boldsymbol{H}^l{}_{pq} + \frac{1}{6}\boldsymbol{H}_{klm}\boldsymbol{H}^k{}_{pq}\boldsymbol{H}_r{}^{lp}\boldsymbol{H}^{rmq}\,. \end{aligned}$$

This action includes all the linear α' corrections for backgrounds of dilaton, metric and NS two form. A general field redefinition

$$\boldsymbol{g}_{ij} \rightarrow \boldsymbol{g}_{ij} + \alpha'\boldsymbol{T}_{ij}\,, \qquad (3.5)$$

$$\boldsymbol{B}_{ij} \rightarrow \boldsymbol{B}_{ij} + \alpha'\boldsymbol{S}_{ij}\,, \qquad (3.6)$$

$$\boldsymbol{\phi} \rightarrow \boldsymbol{\phi} - \alpha'\frac{\boldsymbol{X}}{2}\,, \qquad (3.7)$$

induces a change in $\boldsymbol{L}^{(1)}_{MT}$ of the form [54]

$$\begin{aligned} \Delta\boldsymbol{L} &= -\boldsymbol{T}^{ij}(\boldsymbol{R}_{ij} - \boldsymbol{H}_{ikl}\boldsymbol{H}_j{}^{kl} + 2\nabla_i\nabla_j\boldsymbol{\phi}) + \qquad (3.8)\\ &\quad +(\frac{1}{2}\boldsymbol{T}_i{}^i + \boldsymbol{X})(\boldsymbol{R} - \frac{1}{3}\boldsymbol{H}^2 + 4\nabla^2\boldsymbol{\phi} - 4(\nabla\boldsymbol{\phi})^2) - \nabla_k\boldsymbol{S}_{lm}\boldsymbol{H}^{klm}\,. \end{aligned}$$

where X, S_{ij} and T_{ij} are tensors with appropriate properties and are polynomials of $\boldsymbol{g}_{ij}, \boldsymbol{B}_{ij}, \boldsymbol{\phi}$ and their derivatives.[3] We consider only a class of field redefinition given by

$$\boldsymbol{T}_{ij} = a\boldsymbol{R}_{ij} + b\boldsymbol{H}_{ikl}\boldsymbol{H}_j{}^{kl} + (e-3f)\,\boldsymbol{g}_{ij}\boldsymbol{R} + f\boldsymbol{g}_{ij}\boldsymbol{H}_{klm}\boldsymbol{H}^{klm}\,, \qquad (3.9)$$

$$\boldsymbol{X} + \frac{1}{2}\boldsymbol{T}^i{}_i = (c-3f)\boldsymbol{R} + (d+3f)\boldsymbol{H}_{ijk}\boldsymbol{H}^{ijk}\,, \qquad (3.10)$$

$$\boldsymbol{S}_{ij} = 0\,, \qquad (3.11)$$

where a, b, c, d, e and f are real numbers. This class of field redefinition alters the linear α' corrected action by

$$\Delta\boldsymbol{L} = -a\boldsymbol{R}_{ij}\boldsymbol{R}^{ij} + (c-e)\boldsymbol{R}^2 + (d+e-\frac{c}{3})\boldsymbol{R}\,\boldsymbol{H}^2 - \frac{d}{3}(\boldsymbol{H}^2)^2 \qquad (3.12)$$

[3] To compute $\Delta\boldsymbol{L}$ it is enough to remember that $\boldsymbol{g}^{ij}\delta\boldsymbol{R}_{ij} = (\boldsymbol{\nabla}^i\boldsymbol{\nabla}^j - \boldsymbol{g}^{ij}\Box)\delta\boldsymbol{g}_{ij}$.[55]

$$+(a-b)\boldsymbol{H}^2_{ij}\boldsymbol{R}^{ij}+b\boldsymbol{H}^2_{ij}\boldsymbol{H}^{2ij}\ +O(\nabla\boldsymbol{\phi})\,,$$

where

$$\boldsymbol{H}^2_{ij}=\boldsymbol{H}_{ikl}\boldsymbol{H}_j{}^{kl}\,, \tag{3.13}$$
$$\boldsymbol{H}^2=\boldsymbol{H}_{ijk}\boldsymbol{H}^{ijk}\,, \tag{3.14}$$

and the derivatives of the dilaton are not written to save space. In the forthcoming computations we do not need them. We require the generalisation of the Einstein tensor to be covariantly divergence free for a trivial dilaton. Adding this requirement to the linear α' corrections changes it to the first order Lovelock gravity [56] where $(a, c-e)=(1,\frac{1}{4})$.[4] Thus we set $(a,c)=(1,\frac{1}{4}+e)$ for which the linear α' corrected action reads

$$S\simeq\int d^{10}x\boldsymbol{L} \tag{3.15}$$
$$\boldsymbol{L}=\sqrt{-\det\boldsymbol{g}}e^{-2\phi}(\boldsymbol{R}-\frac{1}{3}\boldsymbol{H}^2+4|\boldsymbol{\nabla\phi}|^2+\alpha'\boldsymbol{L}^{(1)}+\alpha' O(\boldsymbol{\nabla\phi})+O(\alpha'^2)) \tag{3.16}$$
$$\boldsymbol{L}^{(1)}=\frac{1}{4}\boldsymbol{L}_{GB}+(d+\frac{2e}{3}-\frac{1}{12})\boldsymbol{R}\,\boldsymbol{H}^2-\frac{d}{3}(\boldsymbol{H}^2)^2+(1-b)\boldsymbol{H}^2_{ij}\boldsymbol{R}^{ij}+$$
$$+(b-\frac{1}{2})\boldsymbol{H}^2_{ij}\boldsymbol{H}^{2ij}+\frac{1}{6}\boldsymbol{H}_{klm}\boldsymbol{H}^k{}_{pq}\boldsymbol{H}_r{}^{lp}\boldsymbol{H}^{rmq}+\boldsymbol{R}_{klmn}\boldsymbol{H}_p{}^{kl}\boldsymbol{H}^{pmn}$$

where $\boldsymbol{L}_{GB}=\boldsymbol{R}_{ijkl}\boldsymbol{R}^{ijkl}-4\boldsymbol{R}_{ij}\boldsymbol{R}^{ij}+\boldsymbol{R}^2$ is the Gauss-Bonnet term. In work [57] and some follows works the α' corrections were required to not produce new extrema for the bi-linear part of the action describing deviation from flat Minkowski space. This criterion, the no-ghost criterion, is questionable since the new extrema are not perturbative in α'. The criterion we used produces the same results and is independent of the perturbative behaviour of the α' series. However both of these criteria fail to identify a unique action. The MM-criterion [54, 60] which provides a unique action does not produce a horizon for (2.10).

4. Modification of the singularity

We presume that there exists an exact α' background in the large dimensions which in the string frame reads

$$ds_{\text{exact}}=-f(r)dt^2+dr^2+g(r)d\Omega^2_{D-2} \tag{4.1}$$

[4]Lovelock gravity [56] is a generalisation of Einstein-Hilbert action where the generalisation of Einstein tensor G_{ij}: (1) is symmetric in its indices, (2) is a function of the metric and its first two derivatives, (3) is covariantly divergence free. The linear α' corrections can be chosen to satisfy all these conditions [57]. However the higher order α' corrections include also higher derivatives of the metric and can not be rewritten as higher order [58] Lovelock gravity [59].

$$\phi = \phi(r)\,, \qquad\qquad T = T(r)\,, \tag{4.2}$$
$$A_t^{(1)} = A_t^{(1)}(r)\,, \qquad\qquad A_t^{(2)} = A_t^{(2)}(r)\,, \tag{4.3}$$

the large r limits of which are (2.10), (2.11) and (2.12). The number of modified supersymmetry charges [5] of this α' exact background should be the same as the number of SUSY charges of the tree-level background. It is conjectured [32] that this α' exact background has a regular event horizon with isometry group of $AdS_2 \times S^{D-2}$ whose fields in the vicinity of its horizon can be approximated by

$$ds^2 = v_1(-\rho^2 d\tau^2 + \frac{d\rho^2}{\rho^2}) + v_2 d\,\Omega^2_{D-2}\,, \tag{4.4}$$
$$e^{-2\phi(\rho)} = S\,, \tag{4.5}$$
$$T(\rho) = T\,, \tag{4.6}$$
$$F_{t\rho}^{(1)} = f_1\,, \tag{4.7}$$
$$F_{t\rho}^{(2)} = f_2\,, \tag{4.8}$$

where v_1, v_2, S, T, f_1 and f_2 are constant real (S, T are positive) numbers to be fixed by the equations of motion and the behaviour of the fields at infinity. A concrete proof or refutal of this conjecture requires knowing all the α' corrections. Neither the string scattering amplitudes nor the sigma model techniques nor CSFT are practically useful to compute the infinite terms of the α'-expansion series. There exists no other known method capable of producing the full α'-corrected action.[6] Currently the conjecture is supported by

I. Inclusion of only the Gauss-Bonnet action in the induced action allows for the existence of a local horizon geometry whose modified thermodynamical entropy [15, 16, 17] is in agreement with the statistical entropy up to a numerical constant [32].

II. Inclusion of $R_{ijkl}R^{ijkl}$ and the terms needed by SUSY [5, 6, 7, 8, 9, 10, 11, 12, 13, 14] in the four dimensional induced action allows for a local horizon whose modified thermodynamical entropy is in agreement with the statistical entropy [3]. In higher dimensions it is not known which terms should be added to $R_{ijkl}R^{ijkl}$ to maintain SUSY.

The conjecture may be contradicted by :

I. The value of Wald's entropy is not the same for actions related to each other by field redefinition, therefore, which action should be chosen?

[5] In LEEA the supersymmetry algebra is realised as the symmetry of the action, therefore the tree-level algebra may need modification upon the inclusion of the subleading corrections.

[6] There have been attempts to guess a compact form for the α' expansion series of the metric [61, 62].

II. The Gauss-Bonnet action or the supersymmetric version of curvature square terms are not all the linear α' corrections, as pointed out also in [25]. Does the inclusion of all the linear α' corrections allow for the existence of the horizon?

III. Is there a smooth interpolating solution from the horizon toward the asymptotic infinity?

IV. Could the higher order α' corrections be consistently neglected?

First we discuss Wald's formula, next we answer the second question and we try to convince that the answers to the later questions are affirmative.

Wald's formula for entropy has been derived from the Noether current methods [15, 16]. Therefore it is not guaranteed to be invariant under perturbative field redefinitions. Wald's formula for the entropy of a D dimensional static spherical black hole, g_{ij}, is

$$S = \frac{1}{4}\frac{\delta L}{\delta R_{rtrt}}\, g_{tt}\, g_{rr}\, A_{D-2}|_{r=0}\,, \tag{4.9}$$

where L stands for the Lagrangian not including $\sqrt{\det g}$, and A_{D-2} is the area of the horizon, and the radial coordinate is chosen in such a way that the horizon is at $r=0$. Note that $\frac{\delta L}{\delta R_{rtrt}}$ is simply the functional derivative of L with respect to R_{rtrt} holding g_{ij} (and ∇_i) fixed,

$$\frac{\delta L}{\delta R_{rtrt}} = \frac{\partial L}{\partial R_{rtrt}} - \nabla_i \frac{\partial L}{\partial \nabla_i R_{rtrt}} + \cdots\,. \tag{4.10}$$

Consider the Einstein-Hilbert action,

$$S_E = \int d^D x \sqrt{\det g} R\,, \tag{4.11}$$

for which the entropy (4.9) is simplified to $S = \frac{1}{4}A_{D-2}$, as expected. A perturbative metric redefinition,

$$g_{ij} = \hat{g}_{ij}(1 + z\hat{R}) + O(z^2)\,, \tag{4.12}$$

where $\hat{R}$ is the Ricci scalar of $\hat{g}$ and z is the parameter of the perturbation, changes the Einstein-Hilbert action to

$$\hat{S}_E = \int d^D x\, \sqrt{\det \hat{g}}(\hat{R} + \frac{D-2}{2} z\hat{R}^2 + (D-1)z\Box\hat{R})\,. \tag{4.13}$$

The entropy for the redefined action is

$$\hat{S} = \frac{1}{4}(1 + z(D-2)\,\hat{R})\hat{A}_{D-2}|_{r=0}\,, \tag{4.14}$$

which is not the same as the entropy of the initial action,

$$\hat{A}_D(1+z\hat{R})^{\frac{D-2}{2}} = A_D \;\rightarrow\; \hat{S} = (1+\frac{D-2}{2}\,z\,R)S\,. \tag{4.15}$$

This example illustrates that in general the modified Hawking-Bekenstein entropy is not the same for actions related to each other by field redefinitions. Amongst these actions, the choices for which the modified Hawking-Bekenstein entropy is in agreement with the statistical entropy would be preferred. In the following we consider the linear α' corrected action in (3.15) for all values of the field redefinition parameters, (b, d, e, f), and we study if the singular metric (2.10) can be converted to a black hole.

We first obtain the local horizon geometry by solving the equations of motion near the horizon. The equations for the metric and NS two form in ten dimensions derived from (3.15) are second order differential equations and thus after compactification the equations of motion for the gauge fields will remain of second order. For a static background such as (4.1) this implies that

$$\frac{d}{dr}\frac{\delta \boldsymbol{L}}{\delta F^{(i)}_{rt}} = 0 \rightarrow \frac{\delta \boldsymbol{L}}{\delta F^{(i)}_{rt}} = \text{cte} \quad , \quad i \in \{1,2\} \tag{4.16}$$

where $\boldsymbol{L}$ stands for the induced Lagrangian and $F^{(1)}, F^{(2)}$ are the field strengths of $A^{(1)}, A^{(2)}$. We set the value of $\frac{\partial \boldsymbol{L}}{\partial F^{(i)}}$ at infinity, where the α' corrections vanish, equal to its value near the horizon. Near the horizon $\frac{\partial \boldsymbol{L}}{\partial F^{(i)}_{r\tau}}$ is invariant under both time rescaling and re-parametrisation of r. Therefore its value at infinity in the coordinates given by (4.1) is the same as its value near the horizon in the coordinate given by (4.4). Thus (4.16) provides two algebraic equations and relates the near horizon configuration to the fall off of the fields at asymptotic infinity.

In order to obtain the rest of the equations of motion we employ (2.7) to reconstruct the horizon configuration in ten dimensions from the near horizon configuration in the large dimensions (4.4)-(4.8)[7]

$$\begin{aligned} d\boldsymbol{s}^2 &= (-v_1 + 4T^2 f_1^2) r^2 dt^2 + \frac{v_1}{r^2} dr^2 + v_2 d\Omega_{D-2} + T^2 dy^2 + 4 f_1 r T^2 dy dt\,, \\ e^{-2\boldsymbol{\phi}} &= \frac{S}{T}\,, \\ \boldsymbol{B} &= -2 f_2 dt \wedge dr\,. \end{aligned} \tag{4.17}$$

Note that the class of field redefinitions considered in (3.12) includes any field redefinition which produces non-zero terms in the action near the horizon (4.17) and whose metric and B_{ij} equations of motion are second order differential equations.

[7]The compactification of the Gauss-Bonnet action has been done in [63].

Inserting (4.17) in (3.15) and minimising it respect to v_1, v_2, S and T gives four algebraic equations. In general these equations may have multiple solutions. In the following we present one of their solutions. The identification of the near horizon geometry of half BPS backgrounds is an example of the supersymmetric attractor mechanism [64], where the explicit equations of motion are solved rather than the supersymmetric constraints. Solving the equations of motion was first carried out by Ashoke Sen in [32] where only a part of the linear α' corrections were taken into account.

Here we limit ourselves to showing how the equations are solved and provide the details in the appendix. A linear combination of the equations of motion of T and of v_1 factorises

$$\frac{\partial \boldsymbol{L}}{\partial S} = 0 \ \rightarrow \boldsymbol{L} \, = \, 0 \, , \tag{4.18}$$

$$(\frac{1}{T}\frac{\partial \boldsymbol{L}}{\partial T} - 4 f_1^2 \frac{\partial \boldsymbol{L}}{\partial v_1}) \mid_{\boldsymbol{L}=0} \, = \, (T^2 f_1^2 - \frac{v_1^2}{4})(...) \, . \tag{4.19}$$

Eq. (4.19) implies that some of the solutions may be given by

$$f_1 \, = \, \frac{\sqrt{v_1}}{2T} \, . \tag{4.20}$$

From this time on we **set $\boldsymbol{D = 4}$** to write the forthcoming results succinctly. However the same conclusion can be derived in particular for $D = 5$ and for higher dimensions. Eq. (4.20) simplifies the equations of motion of v_1, v_2, S and T and enables one to solve them,

$$f_2 \, = \, \frac{1}{2}\sqrt{v_1}\, x \, T \, , \tag{4.21}$$

$$\frac{v_1}{\alpha} \, = \, \frac{3 + \bar{e}\, x^2}{4} \, , \tag{4.22}$$

$$\frac{v_2}{v_1} \, = \, \frac{4(1 + \bar{e}\, x^2)}{-\bar{e}\, x^4 + (3\bar{e} + 4b - 5)x^2 + 15} \, , \tag{4.23}$$

where x is a root of

$$(-5 - 12d + 12b + 2\bar{e})x^4 + 12(1 - b)x^2 - 18 = 0 \, , \tag{4.24}$$

and $\bar{e}$ is defined by $\bar{e} = 6e + 6d - \frac{1}{2}$. We used x and $\bar{e}$ as different parametrisation of b, d, e to express the near horizon configuration in a more convenient way. Eq. (4.16) provides two other equations

$$\frac{\partial \boldsymbol{L}}{\partial f_1} \, = \, 2(D - 3)N \, , \tag{4.25}$$

$$\frac{\partial \boldsymbol{L}}{\partial f_2} \, = \, 2(D - 3)W \, . \tag{4.26}$$

Using (4.20)-(4.23) in these equations enables one to find T and S

$$T = \sqrt{\frac{N}{W x}} \,. \tag{4.27}$$

$$\frac{v_2}{v_1}\, S = \frac{4\, \sqrt{N\, W\, x\, v_1}}{\alpha'(x^2(2b-1)+3+\frac{4v_1}{v_2})} \tag{4.28}$$

Eq's (4.20), (4.21), (4.22), (4.23), (4.27) and (4.28) identify the near horizon configuration. There exist a set of ranges for the parameters of the field redefinition ambiguity where v_1, v_2, T, S are all positive. It is straightforward to identify these ranges. Here we focus on the subset of the parameters where identity is a root of (4.24) or equivalently $\bar{e} = 6d + \frac{11}{2}$. In this subset T-duality in the y direction (2.6) remains trivial in the sense that interchanging N and W describes T-duality both at asymptotic infinity and near the horizon.[8] For $x = 1$ the near horizon configuration is simplified to

$$v_1 = \frac{17+12d}{8}\, \alpha' \,, \tag{4.29}$$

$$v_2 = \frac{(13+12d)(17+12d)}{4(21+12d+4b)}\, \alpha' \,, \tag{4.30}$$

$$T = \sqrt{\frac{N}{W}} \,, \tag{4.31}$$

$$f_1 = \frac{1}{2}\sqrt{\frac{v_1\, W}{N}} \,, \tag{4.32}$$

$$f_2 = \frac{1}{2}\sqrt{\frac{v_1\, N}{W}} \,, \tag{4.33}$$

$$S = \sqrt{\frac{N\, W}{\alpha'}}\, \frac{21+12d+4b}{2(b+2)\sqrt{34+24d}} \,, \tag{4.34}$$

and thus that for any positive b and d the local horizon exists. The modified Hawking-Bekenstein entropy for all values of b and d is proportional to $\sqrt{NW}$. Perhaps, requiring the modified Hawking-Bekenstein entropy to be equal to the statistical entropy gives an equation amongst whose solutions the one with the largest stretched horizon would be preferred. When the stretched horizon is large the higher order corrections are suppressed outside the horizon and they could be treated as perturbations around the "black hole". We expect that general schemes (4.24), $x \neq 1$, admit solutions for which the modified Hawking-Bekenstein entropy is in agreement with the statistical entropy and $(\frac{v_1}{\alpha'}, \frac{v_2}{\alpha'}) \gg (1,1)$, therefore, the higher order α' corrections could be ignored outside the stretched horizon within these solutions.

[8] In general requiring T-duality to commute with α' corrections identifies corrections to T-duality[65, 66]. One may study α' corrections on various backgrounds to calculate these corrections [67, 68].

Note that there exist field redefinition ambiguities which vanish near the horizon and leave the equations of the metric and NS two-form as second order differential equations. The most general such field redefinition is

$$T_{ij} = c_1\, \boldsymbol{\nabla}_i \boldsymbol{\nabla}_j \boldsymbol{\phi} + c_2 \boldsymbol{g}_{ij} \Box \boldsymbol{\phi} + c_3 \boldsymbol{\nabla}_i \boldsymbol{\phi} \boldsymbol{\nabla}_j \boldsymbol{\phi} + c_4 \boldsymbol{g}_{ij} |\boldsymbol{\nabla} \boldsymbol{\phi}|^2 \tag{4.35}$$

$$X = c_5 \Box \boldsymbol{\phi} + c_6 |\boldsymbol{\nabla} \boldsymbol{\phi}|^2 \tag{4.36}$$

where $c_1, c_2, \cdots, c_6$ are arbitrary real numbers. Ref. [32, 69] have looked for a numerical interpolating solution in one single set of the ambiguity parameters. One should study if there exists any set of values for $b, d, e, f, c_1, \cdots, c_6$ for which a smooth solution interpolates from the near horizon geometry to infinity. This question needs further investigation, however due to the large numbers of the free parameters it is tempting to argue that the interpolating solution exists in general.

5. Conclusions

We have studied the linear α' corrections and field redefinition ambiguities in Heterotic string theory for backgrounds representing a wrapped fundamental string. We have realised these backgrounds as ten dimensional backgrounds compactified to a lower dimensional space-time.

We have added all the linear α' corrections in ten dimensions and taken into account all relevant field redefinitions. We have required the α' corrections to the Einstein tensor to be covariantly divergence free. This requirement has enabled us to rewrite the square of the Riemann tensor as the Gauss-Bonnet Lagrangian keeping some of the field redefinition ambiguity parameters untouched. One may ask if this requirement, similar to the ghost-freedom criterion [70], could be applied to all orders in α'. This question needs further investigation.

We have obtained all the linear α' corrections in lower dimensions through the compactification. We have showed that there exist schemes in which the inclusion of all the linear α' corrections gives rise to a 'local' horizon with geometry $AdS_2 \times S^{D-2}$ and for which the modified Hawking-Bekenstein entropy is in agreement with the statistical entropy. It is surprising that the stretched horizon depends on the scheme, however, considering the scheme dependence of the entropy formula -as we have showed- and the singularity of the tree-level solutions, this dependence is not strange.

We have observed that the α' stretched horizon is large in some schemes. Thus within these schemes the higher order α' corrections can be ignored outside the stretched horizon. We have observed that large classes of field redefinition do not affect the stretched horizon. Therefore at least in one of the subclasses a smooth solution may connect the α' stretched horizon to the fall off of the fields at asymptotic infinity.

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A. The α' corrections near the horizon

The terms of the (3.15) for the ten dimensional near horizon configuration (4.17) read

$$\boldsymbol{L}_{GB} = \frac{4(D-2)(D-3)}{v_1 v_2}(\frac{f_1^2 T^2}{v_1} - 1) + \frac{(D-2)(D-3)(D-4)(D-5)}{v_2^2} \quad (A.1)$$

$$\boldsymbol{R} = -\frac{2}{v_1} + \frac{(D-2)(D-3)}{v_2} + \frac{2f_1^2 T^2}{v_1^2} \quad (A.2)$$

$$\boldsymbol{R}_{klmn}\boldsymbol{H}_p^{\ lm}\boldsymbol{H}^{pkn} = \frac{2f_1^2 f_2^2}{v_1^4} - \frac{2f_2^2}{v_1^3 T^2} \quad (A.3)$$

$$\boldsymbol{H}_{pkl}\boldsymbol{H}^p_{\ mn}\boldsymbol{H}_q^{\ km}\boldsymbol{H}^{qln} = \frac{6f_2^4}{v_1^4 T^4} \quad (A.4)$$

$$\boldsymbol{H}^2 = -\frac{6f_2^2}{v_1^2 T^2} \quad (A.5)$$

$$\boldsymbol{R}^{ij}\boldsymbol{H}^2_{ij} = \frac{4f_2^2}{v_1^3 T^2} - \frac{4f_1^2 f_2^2}{v_1^4} \quad (A.6)$$

$$\boldsymbol{H}^2_{ij}\boldsymbol{H}^{2ij} = \frac{12 f_2^4}{v_1^4 T^4} \quad (A.7)$$

Using the above relations in (3.15) gives an action for v_1, v_2, f_1, f_2, T and S. Minimising this action respect to v_1, v_2, f_1, f_2, T and S gives the equation of motion for the near horizon configuration.

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