Algebraic structure of Lorentz and diffeomorphism anomalies

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Abstract

The Wess-Zumino consistency conditions for Lorentz and diffeomorphism anomalies are discussed by introducing an operator δ which allows to decompose the exterior space-time derivative as a BRS commutator.

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1 Introduction

Since the work of L. Alvarez-Gaumé and E. Witten [1], Lorentz and diffeomorphism anomalies have been the object of continuous investigations and renewed interests. Also if a fully satisfactory understanding of the gravitational phenomena is still lacking, many progress have been done toward a better knowledge of the peculiar features displayed by these anomalies. For instance, according to the analysis of ref. [2] all known diffeomorphism anomalies can be splitted in two groups, called first and second-family, which, by means of a non-polynomial Bardeen-Zumino action [3], can be converted respectively into Weyl and Lorentz anomalies [3, 4, 5, 6, 7, 8]. Moreover, as shown in [1], pure Lorentz anomalies can occur only in $(4n-2)$, $(n \geq 1)$, space-time dimensions.

We emphasize also that a cohomological algebraic set up, known as the $descent-equations$ technique, has been developped by several authors [2, 3, 6, 7, 8, 9] to characterize the gravitational anomalies as non-trivial solutions of the Wess-Zumino consistency conditions [10]. Indeed these conditions, when formulated in terms of the corresponding BRS nilpotent operator s, lead to the characterization of the cohomology of s modulo d, d being the exterior space-time derivative, in the space of local polynomials in the fields and their derivatives. One has then to study the solutions of the equation

$$s\mathcal{A} \ + \ d\mathcal{Q} = 0 \ , \tag{1.1}$$

for some local polynomial $\mathcal{Q}$. $\mathcal{A}$ is said to be non-trivial if

$$\mathcal{A} \neq s\hat{\mathcal{A}} + d\hat{\mathcal{Q}} \ , \tag{1.2}$$

with $\hat{\mathcal{A}}$ and $\hat{\mathcal{Q}}$ local polynomials. In this case the integral of $\mathcal{A}$ on space-time, $\int \mathcal{A}$, yields the anomaly one is looking for. Condition (1.1), due to the relations:

$$s^2 \ = \ d^2 \ = \ sd \ + \ ds \ = 0 \ . \tag{1.3}$$

and to the algebraic Poincaré lemma [11, 9] is easily seen to generate a ladder of equations:

$$
\begin{array}{l}
s\mathcal{Q} \quad + \ d\mathcal{Q}^1 = 0 \\
s\mathcal{Q}^1 \quad + \ d\mathcal{Q}^2 = 0 \\
\quad \ldots\ldots \\
\quad \ldots\ldots \\
s\mathcal{Q}^{k-1} \ + \ d\mathcal{Q}^k = 0 \\
s\mathcal{Q}^k = 0 \ ,
\end{array}
\tag{1.4}
$$

with $\mathcal{Q}^i$ local polynomials in the fields. These equations, as it is well known since several years, can be solved by means of the so-called homotopy "*russian − formula*" [3, 6, 7, 8, 12].

More recently a new way of finding non-trivial solutions of the ladder (1.4) has been proposed by one of the authors [13] and successfully applied to the study of the Yang-Mills cohomology. The method relies on the introduction of an operator δ which allows to decompose the exterior derivative d as a BRS commutator, i.e.:

$$
d \ = \ -[\, s \ , \ \delta \,] \ . \tag{1.5}
$$

It is very easy to show that repeated applications of δ on the polynomial $\mathcal{Q}^k$ which solves the last equation of (1.4) give an explicit expression for the other cocycles $\mathcal{Q}^i$ and for the searched anomaly $\mathcal{A}$. Moreover, as shown in the case of Yang-Mills, these expressions turn out to be cohomologically equivalent to that obtained by the "*russian − formula*".

One has to note also that, actually, the decomposition (1.5) represents one of the most interesting features of the topological field theories. In this case the operator δ is the generator of the topological vector supersymmetry and allows for a complete classification of anomalies and non-trivial observables for both Schwarz and Witten-type topological models [14, 15].

The aim of this work is to extend the analysis of ref. [13] to the gravitational case. In particular we will show that decomposition (1.5) can be used as an alternative tool for an algebraic characterization of the Lorentz and the associated second-family diffeomorphism anomalies.

Let us finish this introduction by making some remarks concerning the field-theory context which will be adopted in the paper.

In what follows we always refer to the gravitational fields, i.e. to the vielbein e, the spin and the Christoffel connections $(\omega,\ \Gamma)$, as external classical fields. The quantum effective action obtained by integrating out the matter fields reduces then to a one-loop expansion.

The functional space which will be used to study the cohomology of the BRS operator is assumed to be the space of the integrated local polynomials in the connections, the Lorentz and diffeomorphism ghosts and their space-time derivatives. This space, also if does not contain the non-polynomial Bardeen-Zumino actions [3], includes all known Lorentz and diffeomorphism anomalies. For what concerns the latters we limit ourselves only to the characterization of the second-family type anomalies which are known to be related with pure Lorentz cocycles [2, 4, 5, 3, 6]. This is due to the fact that, up to our knowledge, it seems rather difficult to establish an algorithm analogue to the "$russian-formula$" for the Weyl trace anomalies. In this case, as discussed by [16] and by [17], the use of the so-called Weyl representation [18] provides an adequate algebraic set up. See also ref. [19] for a quantum mechanical approach and ref. [20] for a classification based on the properties of the conformal Weyl tensor.

We take here the conventional point of view of a field theory locally defined [2]. As shown in [6], global properties can be taken into account by the introduction of a fixed background connection.

Finally, as done in [3, 8], we have adopted the strategy of discussing Lorentz and diffeomorphism anomalies as two separate cohomology problems rather than as a unique one. Let us remark, however, that the expressions we find for the Lorentz cocycles are easily seen to be invariant under the diffeomorphism transformations, due to the fact that the action of the latters on a form-polynomial of maximal space-time degree reduces to a total derivative. The reciprocal property holds also for pure diffeomorphism cocycles which, being expressed in terms of the Christoffel connection, do not contain any Lorentz index.

2 Pure Lorentz anomalies

2.1 Functional idendities

Pure Lorentz anomalies [1], due to the fact that local framing rotations are equivalent to $SO(k)$ (k being the dimension of the euclidean space-time) gauge transformations, can be studied in strict analogy with the corresponding Yang-Mills case. To this purpose, let us introduce the local space $\mathcal{V}$ of form-polynomials [12] in the variables $(\omega,\ d\omega,\ \theta,\ d\theta)$; ω and θ being respectively the one-form spin connection $\omega = \omega_\mu dx^\mu$ and the zero-form Lorentz ghost. The fields ω and θ are Lie algebra valued and are antisymmetric in the Lorentz indices

$$\omega = \frac{1}{2} L^{ab} \omega_{ab} \ , \qquad \theta = \frac{1}{2} L^{ab} \theta_{ab} \ , \tag{2.1}$$

with

$$\omega_{ab} \ = \ -\, \omega_{ba} \ , \qquad \theta_{ab} \ = \ -\, \theta_{ba} \ , \tag{2.2}$$

and $\{L^{ab}\}$ the hermitian generators of $SO(k)$ in some suitable representation. The exterior derivative d is defined by

$$dj_p = dx^\mu \partial_\mu j_p \ , \tag{2.3}$$

for any p-form

$$j_p = \frac{1}{p!} j_{i_1 ....i_p} dx^{i_1} .... dx^{i_p} \ , \tag{2.4}$$

where a wedge product has to be understood. The local space $\mathcal{V}(\omega, d\omega, \theta, d\theta)$ is, as usual, equipped with a grading given by the sum of the form degree and of the ghost number; ω and θ having respectively ghost number zero and one. In general a p-form with ghost number q will be denoted by j_p^q.

The BRS transformations of ω and θ are:

$$\begin{aligned} s\omega &= d\theta - i\{\theta, \omega\} \ , \\ s\theta &= -i\theta^2 \ , \end{aligned} \tag{2.5}$$

with

$$s^2 \ = 0 \ . \tag{2.6}$$

Introducing the two-form Riemann tensor

$$R = d\omega - i\omega^2 \ , \tag{2.7}$$

one has

$$sR = -i[\theta, R] \ , \tag{2.8}$$

and

$$dR = -i[R, \omega] \ , \tag{2.9}$$

which expresses the Bianchi identity.

In what follows we will restrict ourselves to finding the non-trivial solutions of the consistency condition (1.1) belonging to the local space $\mathcal{V}$; i.e. we will assume that the Lorentz anomalies are form-valued polynomials in $(\omega,\ d\omega,\ \theta,\ d\theta)$. This assumption, although if the local space $\mathcal{V}$ does not explicitely contain the vielbein $e = e_\mu dx^\mu$ and the higher order derivatives of ω and θ, is supported by the fact that all known Lorentz anomalies [1, 2, 3, 6, 7, 8, 9] can be expressed in terms of $(\omega,\ d\omega,\ \theta,\ d\theta)$. It is worth to mention that actually, as recently proven by Dubois-Violette et al. [21], the use of the space of form-valued polynomials is not a restriction on the generality of the solutions of the consistency conditions.

To study the cohomology of s and d we use as independent variables the set $(\omega,\ R,\ \theta,\ \rho = d\theta)$; i.e. we replace $d\omega$ with R by using the Bianchi identity (2.9) and we introduce the variable $\rho = d\theta$ to stress the local nature of the condition (1.1). On the local space $\mathcal{V}(\omega, R, \theta, \rho)$ the BRS operator s and the exterior derivative d can be represented as ordinary differential operators:

$$\begin{aligned} s\omega &= \rho - i\{\theta, \omega\} \ , \\ s\theta &= -i\theta^2 \ , \\ sR &= -i[\theta, R] \ , \\ s\rho &= -i[\theta, \rho] \ , \end{aligned} \tag{2.10}$$

and

$$\begin{aligned} d\omega &= R + i\omega^2 \ , \\ d\theta &= \rho \ , \\ dR &= -i[R, \omega] \ , \end{aligned} \tag{2.11}$$

One easily verifies that s and d are of degree one and satisfy

$$s^2 \;=\; d^2 \;=\; sd \;+\; ds \;=0\;. \tag{2.12}$$

2.2 The d and s cohomologies

In this section we briefly recall some useful properties concerning the cohomologies of d and s. Let us begin by showing that, on the local space $\mathcal{V}$, the exterior derivative has vanishing cohomology. Following ref. [13] let us introduce the counting operator $\mathcal{N}$

$$\mathcal{N}\varphi \;=\; \varphi\;, \qquad \varphi=(\omega,\; R,\; \theta,\; \rho)\;, \tag{2.13}$$

according to which the exterior derivative (2.11) decomposes as

$$d \;=\; d^{(0)} \;+\; d^{(1)}\;, \tag{2.14}$$

$$[\,\mathcal{N}\;,\; d^{(\nu)}\,] \;=\; \nu d^{(\nu)}, \qquad \nu=0,1\;, \tag{2.15}$$

with

$$\begin{aligned} d^{(0)}\omega \;&=\; R\;, \\ d^{(0)}\theta \;&=\; \rho\;, \end{aligned} \tag{2.16}$$

and

$$d^{(0)}d^{(0)} \;=\; 0\;. \tag{2.17}$$

Expressions (2.16) show that $d^{(0)}$ has vanishing cohomology; it then follows that also d has vanishing cohomology due to the fact that the cohomology of d is isomorphic to a subspace of the cohomology of $d^{(0)}$ [22].

For what concerns the cohomology of s we have the following result:

s-cohomology [2, 3, 6, 8, 9]

The cohomology of s on $\mathcal{V}(\omega,R,\theta,\rho)$ is given by polynomials in the variables (θ,R) generated by monomials of the form

$$\left(\mathrm{Tr}\,\frac{\theta^{2m+1}}{(2m+1)!}\right)\,\mathcal{P}_{2p+2}(R) \qquad m,p=1,2,....\;, \tag{2.18}$$

where

$$\mathcal{P}_{2p+2}(R) \;=\; \mathrm{Tr}\, R^{p+1} \;\;, \tag{2.19}$$

is the invariant monomial of degree $(2p+2)$.

The indices (m,p) in eq. (2.18) run according to the set of Casimir invariant tensors which can be obtained from traces in the representation $\{L^{ab}\}$. Moreover, as one can immediately see from eq.(2.19), the tensorial group structure of the monomial $\mathcal{P}_{2p+2}(R)$ contains a totally symmetric irreducible invariant Casimir tensor of rank $(p+1)$. Such an invariant tensor exists, however, only if $p=(2n-1)$ [23]. It follows then that pure Lorentz anomalies, besides the abelian case of $SO(2)$, can occur only in $(4n-2)$ space-time dimension [1].

Due to the Bianchi identity (2.9) the invariant monomial $\mathcal{P}_{4n}(R)$ is d-closed

$$d\mathcal{P}_{4n}(R) \; = 0 \;. \tag{2.20}$$

The vanishing of the cohomology of d implies then that $\mathcal{P}_{4n}(R)$ is d-exact:

$$\mathcal{P}_{4n}(R) \;=\; d\mathcal{Q}^{0}_{4n-1} \;. \tag{2.21}$$

Equation (2.21), due to properties (2.12), generates a tower of descent equations:

$$\begin{array}{l} s\mathcal{Q}^{0}_{4n-1} \;+\; d\mathcal{Q}^{1}_{4n-2} = 0 \\ s\mathcal{Q}^{1}_{4n-2} \;+\; d\mathcal{Q}^{2}_{4n-3} = 0 \\ \quad\;\;\cdots\cdots \\ \quad\;\;\cdots\cdots \\ s\mathcal{Q}^{4n-2}_{1} \;+\; d\mathcal{Q}^{4n-1}_{0} = 0 \\ s\mathcal{Q}^{4n-1}_{0} = 0 \;, \end{array} \tag{2.22}$$

where, according to eq.(2.18), the non-trivial solution of the last equation in (2.22) corresponding to the irreducible monomial $\mathcal{P}_{4n}(R)$ is given by the ghost monomial of degree $(4n-1)$:

$$\mathcal{Q}^{4n-1}_{0} \;=\; \mathrm{Tr}\, \frac{\theta^{4n-1}}{(4n-1)!} \;. \tag{2.23}$$

By definition, the cocycles $\mathcal{Q}^1_{4n-2}$ and $\mathcal{Q}^0_{4n-1}$ identify respectively the Lorentz $SO(4n-2)$ anomaly and the Chern-Simons $SO(4n-1)$ term. Consistently, the form part of the Riemann tensor in (2.21) is allowed to have components in extra dimensions.

2.3 Characterization of the solution of the descent equations

To find a solution of the descent equations (2.22) we proceed as in [13] and try to decompose the exterior derivative d as a BRS commutator. This is done by introducing the two operators δ and $\mathcal{G}$

$$\begin{aligned} \delta\theta &= -\omega \ , \\ \delta\rho &= R - i\omega^2 \ , \end{aligned} \tag{2.24}$$

and

$$\begin{aligned} \mathcal{G}\theta &= -R \ , \\ \mathcal{G}\rho &= -i[R,\omega] \ . \end{aligned} \tag{2.25}$$

These operators are respectively of degree zero (δ) and one ($\mathcal{G}$) and obey the following algebraic relations

$$d \ = \ -[\, s \ , \ \delta \,] \ , \tag{2.26}$$

$$[\, d \ , \ \delta \,] \ = \ 2\mathcal{G} \ , \tag{2.27}$$

$$\{\, d \ , \ \mathcal{G} \,\} \ = \ 0 \qquad , \qquad \mathcal{G}\mathcal{G} \ = \ 0 \ , \tag{2.28}$$

$$\{\, s \ , \ \mathcal{G} \,\} \ = \ 0 \qquad , \qquad [\, \mathcal{G} \ , \ \delta \,] = \ 0 \ . \tag{2.29}$$

One sees then that δ and $\mathcal{G}$ give rise to the same algebraic structure as the one already found in [13]. In particular eq.(2.26) shows that the operator δ decomposes the exterior derivative d as a BRS commutator. This feature, as it has been discussed in [13], allows to characterize a solution of the ladder (2.22) in a way equivalent to that of the homotopy "*russian* − *formula*" [3, 6, 7, 8, 12].

Indeed, repeating step by step the same procedure of ref. [13], it is easy to show that a solution of the tower (2.22) is given by

$$\mathcal{Q}^{4n-1}_0 \ = \ \mathrm{Tr}\, \frac{\theta^{4n-1}}{(4n-1)!} \ , \tag{2.30}$$

$$\mathcal{Q}^{4n-1-2p}_{2p} = \frac{\delta^{2p}}{(2p)!}\mathcal{Q}^{4n-1}_0 - \sum_{j=0}^{p-1}\frac{\delta^{2j}}{(2j)!}\Omega^{4n-1-2p+2j}_{2p-2j} , \qquad (2.31)$$

for the even space-time form sector and

$$\mathcal{Q}^{4n-2}_1 = \delta\mathcal{Q}^{4n-1}_0 , \qquad (2.32)$$

$$\mathcal{Q}^{4n-2-2p}_{2p+1} = \frac{\delta^{2p+1}}{(2p+1)!}\mathcal{Q}^{4n-1}_0 - \sum_{j=0}^{p-1}\frac{\delta^{2j+1}}{(2j+1)!}\Omega^{4n-1-2p+2j}_{2p-2j} , \qquad (2.33)$$

for the odd sector and $p = 1, 2, ..., (2n-1)$.

The Ω expressions in eqs.(2.31), (2.33) are, as in [13], solutions of a second tower of descent equations which originates from the algebraic relations (2.28)-(2.29) and from the s-cohomology (2.18). They read:

$$\begin{array}{lcl} \mathcal{G}\left(\mathrm{Tr}\,\frac{\theta^{4n-1}}{(4n-1)!}\right) & = & s\Omega^{4n-3}_2 \\ \mathcal{G}\Omega^{4n-3}_2 + s\Omega^{4n-5}_4 & = & 0 \\ \mathcal{G}\Omega^{4n-5}_4 + s\Omega^{4n-7}_6 & = & 0 \\ \quad ...... & & \\ \quad ...... & & \\ \mathcal{G}\Omega^{3}_{4n-4} + s\Omega^{1}_{4n-2} & = & 0 \ . \end{array} \qquad (2.34)$$

and

$$\mathcal{G}\Omega^1_{4n-2} = (const)\mathcal{P}_{4n}(R) , \qquad (2.35)$$

where $\mathcal{P}_{4n}(R)$ is the invariant polynomial of eq. (2.19) and the $(const)$ is an easily computed algebraic factor.

Finally, let us recall that eqs.(2.30)-(2.33) identify a class of solutions which, usually, is not the most general one. However, as it is well known [12], once a particular solution has been found, the search of the most general solution reduces essentially to a problem of BRS local cohomology instead of a modulo-d one. The latter is easily disentangled by using the result (2.18) and the algebraic structure (2.26)-(2.29).

2.4 Some examples

The purpose of this section is to apply the previous construction to discuss some explicit examples. We will consider, in particular, the cases of $n = 1, 2$ which correspond respectively to the two and the six-dimensional Lorentz anomalies as well as to the three and the seven-dimensional Chern-Simons terms.

The case n=1

In this case, relevant for the $SO(3)$ Chern-Simons term, the descent equations (2.22) read:

$$\begin{array}{l} s\mathcal{Q}^0_3 \ + \ d\mathcal{Q}^1_2 = 0 \\ s\mathcal{Q}^1_2 \ + \ d\mathcal{Q}^2_1 = 0 \\ s\mathcal{Q}^2_1 \ + \ d\mathcal{Q}^3_0 = 0 \\ s\mathcal{Q}^3_0 = 0 \ , \end{array} \tag{2.36}$$

According to eqs. (2.30)-(2.33) and (2.34) the corresponding solution is given by

$$\mathcal{Q}^0_3 \ = \ \frac{1}{3!}\delta\delta\delta\mathcal{Q}^3_0 \ - \ \delta\Omega^1_2 \ , \tag{2.37}$$

$$\mathcal{Q}^1_2 \ = \ \frac{1}{2}\delta\delta\mathcal{Q}^3_0 \ - \ \Omega^1_2 \ , \tag{2.38}$$

$$\mathcal{Q}^2_1 \ = \ \delta\mathcal{Q}^3_0 \ , \tag{2.39}$$

with

$$\mathcal{Q}^3_0 \ = \ \mathrm{Tr}\,\frac{\theta^3}{3!} \ , \tag{2.40}$$

and Ω^1_2 solution of the equation

$$\mathcal{G}\left(\mathrm{Tr}\,\frac{\theta^3}{3!}\right) \ = \ s\Omega^1_2 \ . \tag{2.41}$$

Eq. (2.41) is easily solved with Ω^1_2:

$$\Omega^1_2 \ = \ \frac{i}{2}\mathrm{Tr}\,R\theta \ . \tag{2.42}$$

Equations (2.37)-(2.39) become then:

$$\mathcal{Q}^0_3 \;=\; \frac{1}{2}\mathrm{Tr}\left(iR\omega - \frac{\omega^3}{3}\right)\;, \tag{2.43}$$

$$\mathcal{Q}^1_2 \;=\; \frac{1}{2}\mathrm{Tr}\left(\omega\omega\theta - iR\theta\right) \;=\; -\frac{i}{2}\mathrm{Tr}\,\theta d\omega\;, \tag{2.44}$$

$$\mathcal{Q}^2_1 \;=\; -\frac{1}{2}\mathrm{Tr}\,\omega\theta\theta\;. \tag{2.45}$$

In particular eq.(2.43) gives the familiar $SO(3)$ Chern-Simons form. It is interesting to note that, in spite of the fact that $\mathrm{Tr}\,\frac{\theta^3}{3}$ is identically zero for the Lorentz group $SO(2)$, the cocycle $\mathcal{Q}^1_2$ in eq.(2.44), when referred to $SO(2)$, identifies also the two-dimensional Lorentz anomaly. In this case, indeed, the BRS transformations (2.5) become abelian, i.e.:

$$\begin{aligned} s\omega_{12} &= d\theta_{12}\;, \\ s\theta_{12} &= 0\;, \end{aligned} \tag{2.46}$$

ω_{12} and θ_{12} being the unique non-zero components of the spin connection and of the Lorentz ghost. The corresponding cocycle, $\mathcal{A}_{abelian}$, is then

$$\mathcal{A}_{abelian} = \theta_{12} d\omega_{12}\;, \tag{2.47}$$

and, as shown by explicit computations [24], turns out to be the most general expression for the two-dimensional Lorentz anomaly.

The case n=2

In this example, relevant for the six-dimensional Lorentz anomaly and for the seven-dimensional Chern-Simons term, the tower (2.22) takes the form:

$$\begin{aligned} &s\mathcal{Q}^0_7 \;+\; d\mathcal{Q}^1_6 = 0 \\ &s\mathcal{Q}^1_6 \;+\; d\mathcal{Q}^2_5 = 0 \\ &s\mathcal{Q}^2_5 \;+\; d\mathcal{Q}^3_4 = 0 \\ &s\mathcal{Q}^3_4 \;+\; d\mathcal{Q}^4_3 = 0 \\ &s\mathcal{Q}^4_3 \;+\; d\mathcal{Q}^5_2 = 0 \\ &s\mathcal{Q}^5_2 \;+\; d\mathcal{Q}^6_1 = 0 \\ &s\mathcal{Q}^6_1 \;+\; d\mathcal{Q}^7_0 = 0 \\ &s\mathcal{Q}^7_0 = 0\;. \end{aligned} \tag{2.48}$$

From eqs. (2.30)-(2.33) a class of solutions is given by

$$\mathcal{Q}^0_7 \;=\; \frac{1}{7!}\delta^7\mathcal{Q}^7_0 \;-\; \frac{1}{5!}\delta^5\Omega^5_2 \;-\; \frac{1}{3!}\delta^3\Omega^3_4 \;-\; \delta\Omega^1_6 \;, \tag{2.49}$$

$$\mathcal{Q}^1_6 \;=\; \frac{1}{6!}\delta^6\mathcal{Q}^7_0 \;-\; \frac{1}{4!}\delta^4\Omega^5_2 \;-\; \frac{1}{2!}\delta^2\Omega^3_4 \;-\; \Omega^1_6 \;, \tag{2.50}$$

$$\mathcal{Q}^2_5 \;=\; \frac{1}{5!}\delta^5\mathcal{Q}^7_0 \;-\; \frac{1}{3!}\delta^3\Omega^5_2 \;-\; \delta\Omega^3_4 \;\;, \tag{2.51}$$

$$\mathcal{Q}^3_4 \;=\; \frac{1}{4!}\delta^4\mathcal{Q}^7_0 \;-\; \frac{1}{2!}\delta^2\Omega^5_2 \;-\; \Omega^3_4 \;\;, \tag{2.52}$$

$$\mathcal{Q}^4_3 \;=\; \frac{1}{3!}\delta^3\mathcal{Q}^7_0 \;-\; \delta\Omega^5_2 \;\;, \tag{2.53}$$

$$\mathcal{Q}^5_2 \;=\; \frac{1}{2!}\delta^2\mathcal{Q}^7_0 \;-\; \Omega^5_2 \;\;, \tag{2.54}$$

$$\mathcal{Q}^6_1 \;=\; \delta\mathcal{Q}^7_0 \;\;, \tag{2.55}$$

and

$$\mathcal{Q}^7_0 \;=\; \mathrm{Tr}\,\frac{\theta^7}{7!} \;. \tag{2.56}$$

The Ω-cocycles are, according to eqs. (2.34), solutions of the ladder

$$\begin{aligned} \mathcal{G}\left(\mathrm{Tr}\,\tfrac{\theta^7}{7!}\right) &\;=\; s\Omega^5_2 \\ \mathcal{G}\Omega^5_2 \;+\; s\Omega^3_4 &\;=\; 0 \\ \mathcal{G}\Omega^3_4 \;+\; s\Omega^1_6 &\;=\; 0 \;, \end{aligned} \tag{2.57}$$

and read

$$\Omega^5_2 \;=\; \frac{i}{6!}\mathrm{Tr}\,\left(R\theta^5\right) \;, \tag{2.58}$$

$$\Omega^3_4 \;=\; \frac{1}{6!}\mathrm{Tr}\,\left(2R^2\theta^3 + R\theta R\theta^2\right) \;, \tag{2.59}$$

$$\Omega^1_6 \;=\; -\,i\frac{5}{6!}\mathrm{Tr}\,\left(R^3\theta\right) \;. \tag{2.60}$$

$\mathcal{Q}^0_7$ and $\mathcal{Q}^1_6$ are computed to be

$$\mathcal{Q}^0_7 \;=\; -\frac{5}{6!}\mathrm{Tr}\,\left(iR^3\omega - \frac{2}{5}R^2\omega^3 - \frac{1}{5}R\omega R\omega^2 - \frac{i}{5}R\omega^5 + \frac{1}{35}\omega^7\right) \;, \tag{2.61}$$

$$
\begin{aligned}
\mathcal{Q}^1_6 \;=\; & \tfrac{5}{6!}\,\mathrm{Tr}\,(\; iR^3\theta - \tfrac{2}{5}(R^2\omega^2\theta + R^2\omega\theta\omega + R^2\theta\omega^2)\;) \\
& -\tfrac{1}{5}\mathrm{Tr}\,(\; R\omega R\omega\theta + R\omega R\theta\omega + R\theta R\omega^2\;) \;+\; \tfrac{1}{5}\mathrm{Tr}\,\omega^6\theta \\
& -\tfrac{i}{5}\mathrm{Tr}\,(\; R\omega^4\theta + R\omega^3\theta\omega + R\omega^2\theta\omega^2 + R\omega\theta\omega^3 + R\theta\omega^4\;)\;,
\end{aligned}
\tag{2.62}
$$

and give respectively the seven-dimensional Chern-Simons form and the six-dimensional Lorentz anomaly. One has to note that, in this case, expressions (2.61)-(2.62) do not identify the most general solution of the descent equations (2.48). This is due to the fact that the local cohomology of s is non-vanishing in the sector with ghost number three and form-degree four. Indeed from eq.(2.18) it follows that the factorized monomial

$$
\mathcal{B}^3_4 = \left(\mathrm{Tr}\,R^2\right)\left(\mathrm{Tr}\,\frac{\theta^3}{3}\right)\;, \tag{2.63}
$$

belongs to the local cohomology of s and, when inserted in the ladder (2.48), gives origin to the sub-tower:

$$
\begin{aligned}
& s\mathcal{B}^0_7 \;+\; d\mathcal{B}^1_6 = 0 \\
& s\mathcal{B}^1_6 \;+\; d\mathcal{B}^2_5 = 0 \\
& s\mathcal{B}^2_5 \;+\; d\mathcal{B}^3_4 = 0 \\
& s\mathcal{B}^3_4 = 0\;.
\end{aligned}
\tag{2.64}
$$

These equations, thanks to the fact that $\mathcal{B}^3_4$ is a factorized cocycle, are easily solved by using the previous results for the case $n = 1$ (eqs. (2.43) - (2.45)). They yield the following solution:

$$
\mathcal{B}^0_7 \;=\; \frac{1}{2}\left(\mathrm{Tr}\,R^2\right)\left(\mathrm{Tr}\,(iR\omega - \frac{\omega^3}{3})\right)\;, \tag{2.65}
$$

$$
\mathcal{B}^1_6 \;=\; \frac{1}{2}\left(\mathrm{Tr}\,R^2\right)\left(\mathrm{Tr}\,(\omega\omega\theta - iR\theta)\right)\;, \tag{2.66}
$$

$$
\mathcal{B}^2_5 \;=\; -\frac{1}{2}\left(\mathrm{Tr}\,R^2\right)\left(\mathrm{Tr}\,\omega\theta\theta\right)\;. \tag{2.67}
$$

One sees that $\mathcal{B}^1_6$ and $\mathcal{B}^0_7$ are given respectively by the factorized product of $\mathrm{Tr}\,R^2$ with the two-form Lorentz anomaly and the three-form Chern-Simons term; the only difference being that the trace is now taken over a representation of $SO(6)$ (resp. $SO(7)$) instead of $SO(2)$

(resp. $SO(3)$). The most general algebraic non-trivial solution for the six-dimensional Lorentz anomaly is given then by the sum of the irreducible element (2.62) and of the factorized term in eq.(2.66). This example illustrates in a clear way a general feature of the descent equations (2.22). It is easy to check indeed that the most general solution of the ladder (2.22) is usually given by the sum of an irreducible term (corresponding to the ghost monomial (2.23)) and of the whole set of factorized elements which are solutions of all the non-trivial sub-towers allowed by the local cohomology of s [12].

3 Pure diffeomorphism anomalies

3.1 Generalities

To characterize the diffeomorphism anomalies let us begin by specifying the set of variables and the functional space the nilpotent BRS operator acts upon. Following [2, 3, 6, 7, 8] we will adopt as fundamental variable the particular combination of the spin connection ω and the vielbein e called Christoffel connection [1]:

$$\Gamma = e^{-1}de + e^{-1}\omega e \ , \qquad \Gamma^{\rho}_{\mu\lambda} = \Gamma^{\rho}_{\lambda\mu} \ . \tag{3.1}$$

Denoting with ξ^{μ} the ghost field associated with the infinitesimal diffeomorphism transformations, the corresponding BRS operator reads:

$$\begin{aligned} & s\Gamma^{\rho}_{\lambda\mu} = \xi^{\sigma}\partial_{\sigma}\Gamma^{\rho}_{\lambda\mu} + (\partial_{\lambda}\xi^{\sigma})\Gamma^{\rho}_{\sigma\mu} + (\partial_{\mu}\xi^{\sigma})\Gamma^{\rho}_{\lambda\sigma} - (\partial_{\sigma}\xi^{\rho})\Gamma^{\sigma}_{\lambda\mu} - \partial_{\lambda}\partial_{\mu}\xi^{\rho} \ , \\ & s\xi^{\mu} = \xi^{\sigma}\partial_{\sigma}\xi^{\mu} \ , \end{aligned} \tag{3.2}$$

and

$$s^2 = 0 \ . \tag{3.3}$$

Consequently, the functional space the operator s acts upon is assumed to be the space $\mathcal{F}$ of the polynomials in the variables (Γ, ξ) and their space-time derivatives. This functional local space turns out to be large enough to contain all the diffeomorphism anomalies; both

[1] For simplicity we consider here only the case of vanishing torsion.

those related to the pure Lorentz anomalies and those related to Weyl anomalies.

For a better understanding of this point let us quote an important result, valid for any space-time dimension, on the general form of the diffeomorphism anomalies.

Diffeomorphism anomalies [2]

On the space $\mathcal{F}(\Gamma, \xi)$ the most general diffeomorphism anomaly $\mathcal{A}_{diff}$ has the form

$$\mathcal{A}_{diff} = \int (b\partial_\mu \xi^\mu + b^{\mu\nu}_\sigma \partial_\mu \partial_\nu \xi^\sigma) \ , \tag{3.4}$$

where b is a scalar density and cannot be written as a total derivative and $b^{\mu\nu}_\sigma$ is a tensor under linear GL-transformations. [2]

The coefficients b and $b^{\mu\nu}_\sigma$ define the so-called first and second-family of diffeomorphism cocycles and are associated respectively with Weyl and Lorentz anomalies [2, 3, 4, 5].

Being interested only in the latter anomalies we focus on the analysis of the second-family; we proceed then to the algebraic characterization of the coefficient $b^{\mu\nu}_\sigma$. We recall also that, up to the present time, all known diffeomorphism anomalies of the second-family type belong to a local space which, as in the Lorentz case, consists of polynomials of differential forms [2, 3, 6, 7, 8, 9] built with the Christoffel connection and the Riemann tensor.

We introduce then the space of the form-valued polynomials in the variables $(\Gamma^\rho_\mu,\ R^\rho_\mu,\ \Lambda^\rho_\mu,\ \lambda^\rho_\mu)$ where:

i) Γ^ρ_μ denotes the one-form connection

$$\Gamma^\rho_\mu = \Gamma^\rho_{\lambda\mu} dx^\lambda \ , \tag{3.5}$$

[2] Diffeomorphism transformations (3.2) reduce to linear GL-transformations in the special case of $\xi^\mu(x) = \alpha^\mu_\sigma x^\sigma$, with α constant parameters.

$ii)$ R^ρ_μ is the two-form Riemann tensor

$$R^\rho_\mu = d\Gamma^\rho_\mu - \Gamma^\rho_\sigma \Gamma^\sigma_\mu, \tag{3.6}$$

$iii)$ Λ^ρ_μ is the zero-form

$$\Lambda^\rho_\mu = -\partial_\mu \xi^\rho \ , \tag{3.7}$$

$iv)$ λ^ρ_μ is the one-form

$$\lambda^\rho_\mu = \ - dx^\sigma \partial_\sigma \Lambda^\rho_\mu \ = \ - d\Lambda^\rho_\mu \ , \tag{3.8}$$

and d is the ordinary exterior space-time derivative (2.3).

The diffeomorphism anomalies of the second-family will be defined then as local polynomials $\mathcal{A}^1_{max}$ in the variables $(\Gamma,\ R,\ \Lambda,\ \lambda)$ of ghost number one and maximal space-time form-degree which are non-trivial solutions of the consistency condition

$$s\mathcal{A}^1_{max} + d\mathcal{A}^2_{max-1} = 0 \ , \tag{3.9}$$

where, according to (3.2), the action of the BRS operator on the space of form-polynomials in $(\Gamma,\ R,\ \Lambda,\ \lambda)$ is defined as

$$\begin{aligned}
s\Gamma^\rho_\mu &= \lambda^\rho_\mu + \Gamma^\rho_\sigma \Lambda^\sigma_\mu - \Gamma^\sigma_\mu \Lambda^\rho_\sigma + \mathcal{L}_\xi \Gamma^\rho_\mu \ , \\
sR^\rho_\mu &= \Lambda^\rho_\sigma R^\sigma_\mu - \Lambda^\sigma_\mu R^\rho_\sigma + \mathcal{L}_\xi R^\rho_\mu \ , \\
s\Lambda^\rho_\mu &= \Lambda^\rho_\sigma \Lambda^\sigma_\mu + \mathcal{L}_\xi \Lambda^\rho_\mu \ , \\
s\lambda^\rho_\mu &= \Lambda^\rho_\sigma \lambda^\sigma_\mu - \Lambda^\sigma_\mu \lambda^\rho_\sigma + \mathcal{L}_\xi \lambda^\rho_\mu \ .
\end{aligned} \tag{3.10}$$

The symbol $\mathcal{L}_\xi$ in eqs.(3.10) denotes the ordinary Lie derivative, taken with respect to the ghost parameter ξ, which acts only on the form-indices of $(\Gamma,\ R,\ \Lambda,\ \lambda)$ as for instance:

$$\mathcal{L}_\xi \Gamma^\rho_\mu = \xi^\sigma \partial_\sigma \Gamma^\rho_\mu + \partial_\lambda \xi^\sigma \Gamma_{\sigma\mu} dx^\lambda \equiv \xi^\sigma \partial_\sigma \Gamma^\rho_\mu - \Lambda^\sigma_\lambda \Gamma_{\sigma\mu} dx^\lambda \ . \tag{3.11}$$

Notice that only one space-time index of the Christoffel connection (3.1) has been referred to a form-index. The BRS transformations (3.10) (apart the term of Lie derivative) have thus the same structure of a gauge transformation with Λ^ρ_μ as gauge parameter, Γ^ρ_μ as gauge connection and with the Riemann tensor R^ρ_μ as the ordinary Yang-Mills field strength.

3.2 Decomposition of the BRS operator

On the local space $(\Gamma,\ R,\ \Lambda,\ \lambda)$ the BRS operator of eqs.(3.10) naturally decomposes as [3]:

$$s = s_0 + \mathcal{L}_\xi \ , \tag{3.12}$$

with

$$\begin{aligned} s_0 = &(\lambda^\rho_\mu + \Gamma^\rho_\sigma\Lambda^\sigma_\mu - \Gamma^\sigma_\mu\Lambda^\rho_\sigma)\frac{\partial}{\partial\Gamma^\rho_\mu} + \Lambda^\rho_\sigma\Lambda^\sigma_\mu\frac{\partial}{\partial\Lambda^\rho_\mu} \\ &+(\Lambda^\rho_\sigma R^\sigma_\mu - \Lambda^\sigma_\mu R^\rho_\sigma)\frac{\partial}{\partial R^\rho_\mu} + (\Lambda^\rho_\sigma\lambda^\sigma_\mu - \Lambda^\sigma_\mu\lambda^\rho_\sigma)\frac{\partial}{\partial\lambda^\rho_\mu} \ , \end{aligned} \tag{3.13}$$

$$s_0 s_0 = 0 \ , \tag{3.14}$$

One easily sees that the nilpotent operator s_0 in eq.(3.13) has exactely the same form of the corresponding BRS-Lorentz operator (2.10); Γ and Λ playing the role of the spin-connection ω and of the Lorentz ghost θ.

It should be noted also that the action of the Lie derivative $\mathcal{L}_\xi$ in (3.12) on a form-polynomial of maximal space-time degree reduces to a total derivative [3]; i.e.

$$\mathcal{L}_\xi\Omega^p_{max} = d\Omega^{p+1}_{max-1} \ , \tag{3.15}$$

Ω^p_{max} denoting a form-polynomial of maximal degree and p ghost number. Now, according to our previous definition, the diffeomorphism anomalies of the second-family type $\mathcal{A}^1_{max}$ are form-polynomials of maximal space-time degree which are non-trivial solutions of the consistency condition (3.9). It follows then:

$$\mathcal{L}_\xi\mathcal{A}^1_{max} = d\mathcal{B}^2_{max-1} \ , \tag{3.16}$$

for some local form-polynomial $\hat{\mathcal{B}}^2_{max-1}$. Equation (3.16) tells us that, on the space of forms of maximal degree, the cohomology of s modulo d and that of s_0 modulo d are in one to one correspondence; the difference between the two operators being a total derivative. We can thus replace s by s_0 in the consistency equation (3.9) and characterize $\mathcal{A}^1_{max}$ as a non-trivial element of the cohomology of s_0 modulo d.

Moreover, to identify the cohomolgy os s_0 modulo d, we can use the same algebraic procedure of the corresponding Lorentz case. Indeed, one checks that the following algebraic relations hold:

$$d \ = \ -[\ s_0\ ,\ \delta_0\ ]\ , \tag{3.17}$$

$$[\ d\ ,\ \delta_0\ ] \ = \ 2\mathcal{G}_0\ , \tag{3.18}$$

$$\{\ d\ ,\ \mathcal{G}_0\ \} \ = \ 0 \qquad , \qquad \mathcal{G}_0\mathcal{G}_0 \ = \ 0\ , \tag{3.19}$$

$$\{\ s_0\ ,\ \mathcal{G}_0\ \} \ = \ 0 \qquad , \qquad [\ \mathcal{G}_0\ ,\ \delta_0\ ] = \ 0\ , \tag{3.20}$$

where, in analogy with expressions (2.11), (2.24) and (2.25), the operators d, δ_0 and $\mathcal{G}_0$ are given by

$$d = (R^\rho_\mu + \Gamma^\rho_\sigma\Gamma^\sigma_\mu)\frac{\partial}{\partial\Gamma^\rho_\mu} - \lambda^\rho_\mu\frac{\partial}{\partial\Lambda^\rho_\mu} + (\Gamma^\rho_\sigma R^\sigma_\mu - \Gamma^\sigma_\mu R^\rho_\sigma)\frac{\partial}{\partial R^\rho_\mu}\ , \tag{3.21}$$

$$\delta_0 = \Gamma^\rho_\mu\frac{\partial}{\partial\Lambda^\rho_\mu} + (R^\rho_\mu - \Gamma^\rho_\sigma\Gamma^\sigma_\mu)\frac{\partial}{\partial\lambda^\rho_\mu}\ , \tag{3.22}$$

and

$$\mathcal{G}_0 = R^\rho_\mu\frac{\partial}{\partial\Lambda^\rho_\mu} + (\Gamma^\rho_\sigma R^\sigma_\mu - \Gamma^\sigma_\mu R^\rho_\sigma)\frac{\partial}{\partial\lambda^\rho_\mu}\ . \tag{3.23}$$

Expressions (3.13)-(3.14) and (3.21) allow to easily adapt the results (2.14)-(2.17) and (2.18) on the Lorentz cohomology to the case of the nilpotent operator s_0. In particular, the exterior derivative (3.21) turns out to have vanishing cohomology.

It becomes apparent then that starting with a Lorentz anomaly we can immediately compute the corresponding second-family diffeomorphism anomaly; it is sufficient to replace the spin connection ω with the one-form Christoffel connection Γ and the Lorentz ghost θ with the variable Λ. As an example, the two dimensional diffeomorphism anomaly is, according to expressions (2.44)-(2.47), given by

$$\int \Lambda^\rho_\mu d\Gamma^\mu_\rho\ . \tag{3.24}$$

As in the Lorentz case, the operator δ_0, and $\mathcal{G}_0$ give a simple procedure for solving the cohomology of s_0 modulo d and finding the second-family diffeomorphism anomalies.

3.3 A four-dimensional example

For a better understanding of the algebraic relations (3.17)-(3.20) let us conclude this section by discussing the construction of a non-trivial four-dimensional diffeomorphism cocycle in the space of local form-polynomials; this example will give us the possibility of underlining the importance of the general result (3.4) and of evidentiating the difference between the Lorentz and the diffeomorphism transformations.

We emphasize indeed that, as long as one is concerned only with diffeomorphism cocycles, the result (3.4), being valid in any space-time dimension, does not forbid the existence of non-trivial solutions of the consistency condition (3.9) living in a space-time whose dimensions do not allow for a Lorentz counterpart. This is the case, for instance, of the four dimensional space-time.

Let us look then for a non-trivial solution of the consistency condition

$$s_0\mathcal{A}^1_4 + d\mathcal{A}^2_3 = 0 \ , \tag{3.25}$$

where s_0 and d are the operators given in eqs.(3.13), (3.21) and $\mathcal{A}^1_4$ and $\mathcal{A}^2_3$ are form-polynomials. Equation (3.25), due to the vanishing of the cohomology of d, generates the tower

$$\begin{aligned} &s_0\mathcal{A}^2_3 \ + \ d\mathcal{A}^3_2 = 0 \\ &s_0\mathcal{A}^3_2 \ + \ d\mathcal{A}^4_1 = 0 \\ &s_0\mathcal{A}^4_1 \ + \ d\mathcal{A}^5_0 = 0 \\ &s_0\mathcal{A}^5_0 = 0 \ . \end{aligned} \tag{3.26}$$

According to the local cohomolgy of s_0, $\mathcal{A}^5_0$ is given by

$$\mathcal{A}^5_0 = \frac{1}{5!}\Lambda^5 \equiv \frac{1}{5!}\Lambda^\mu_\nu\Lambda^\nu_\sigma\Lambda^\sigma_\tau\Lambda^\tau_\rho\Lambda^\rho_\mu \ . \tag{3.27}$$

As in the Lorentz case, a solution of eqs.(3.25)-(3.26) is easily obtained by acting with the operator δ_0 of eq.(3.22) on the expression (3.27). One easily finds:

$$\mathcal{A}^1_4 \ = \ \frac{1}{4!}\delta_0\delta_0\delta_0\delta_0\mathcal{A}^5_0 \ - \ \frac{1}{2}\delta_0\delta_0\Omega^3_2 \ - \ \Omega^1_4 \ , \tag{3.28}$$

$$\mathcal{A}^2_3 \;=\; \frac{1}{3!}\delta_0\delta_0\delta_0\mathcal{A}^5_0 \;-\; \delta_0\Omega^3_2 \;\;, \tag{3.29}$$

$$\mathcal{A}^3_2 \;=\; \frac{1}{2}\delta_0\delta_0\mathcal{A}^5_0 \;-\; \Omega^3_2 \;\;, \tag{3.30}$$

$$\mathcal{A}^4_1 \;=\; \delta_0\mathcal{A}^5_0 \;\;, \tag{3.31}$$

where, according to the tower (2.34)-(2.35), Ω^3_2 and Ω^1_4 are solutions of

$$\begin{array}{l} \mathcal{G}_0\mathcal{A}^5_0 \;=\; s_0\Omega^3_2 \\ \mathcal{G}_0\Omega^3_2 \;+\; s_0\Omega^1_4 \;=\; 0 \;, \end{array} \tag{3.32}$$

and computed to be:

$$\Omega^3_2 \;=\; -\frac{1}{4!}R^\mu_\nu\Lambda^\nu_\sigma\Lambda^\sigma_\rho\Lambda^\rho_\mu \;, \tag{3.33}$$

$$\Omega^1_4 \;=\; -\frac{2}{4!}R^\mu_\nu R^\nu_\sigma\Lambda^\sigma_\mu \;. \tag{3.34}$$

In particular, the cocycle $\mathcal{A}^1_4$ is given by

$$\begin{array}{ll} \mathcal{A}^1_4 & = \frac{1}{4!}(\Gamma\Gamma\Gamma\Gamma\Lambda + 2RR\Lambda + R\Gamma\Gamma\Lambda + R\Gamma\Lambda\Gamma + R\Lambda\Gamma\Gamma) \\ & \equiv -\frac{2}{4!}\Lambda \; d(\Gamma d\Gamma - \frac{1}{2}\Gamma\Gamma\Gamma) \;, \end{array} \tag{3.35}$$

and coincides, modulo a d-coboundary, with that of ref. [2, 4, 5]. Expression (3.35) is a non-trivial solution of the consistency condition (3.25) in the space of the form-polynomials which does not possess a Lorentz counterpart. This is due to the fact that the analogue of the ghost monomial (3.27) in the $SO(4)$-Lorentz case identically vanishes due to the antisymmetric properties of θ and to the absence of a third order symmetric invariant Casimir tensor [23].

Finally, let us recall that expression (3.35), also if cohomologically non-trivial in the local space of form-polynomials, does not forbid the construction of an effective quantum action which preserves both Lorentz and diffeomorphism invariances. It is known indeed that, using the logarithm of the vielbein as the Goldstone boson field [3], the cocycle (3.35) can be mapped into zero by a non-polynomial Bardeen-Zumino [2] action; according to the well established result that non-trivial anomalies can arise only in $(4n-2)$ space-time dimensions [1].

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References

[1] L. Alvarez-Gaumé and E. Witten, *Nucl. Phys.* B234 (1983) 269;

[2] L. Bonora, P. Pasti and M. Tonin, *J. Math. Phys.* 27 (1986) 2259;

[3] W. A. Bardeen and B. Zumino, *Nucl. Phys.* B244 (1984) 421;

[4] L. Bonora, P. Pasti and M. Tonin, *Phys. Lett.* B149 (1984) 346;

[5] L. Bonora, P. Pasti and M. Bregola, *Class. Quantum Grav.* 3 (1986) 635;

[6] F. Langouche, T. Schucker and R. Stora, *Phys. Lett.* B145 (1984) 342;
R. Stora , *Algebraic structure of chiral anomalies*; lecture given at the GIFT seminar, 1-8 june 1985, Jaca, Spain; preprint LAPP-TH-143;
T. Schucker, *Comm. Math. Phys.* 109 (1987) 167;

[7] L. Baulieu and J. Thierry-Mieg, *Phys. Lett.* B145 (191984) 53;

[8] L. Alvarez-Gaumé and P. Ginsparg, *Ann. Phys. (N.Y.)* 161 (1985) 423;

[9] F. Brandt, N. Dragon and M. Kreuzer, *Nucl. Phys.* B340 (1990) 187;

[10] J. Wess and B. Zumino, *Phys. Lett.* B37 (1971) 95;

[11] L. Bonora and P. Cotta-Ramusino, *Comm. Math. Phys.* 87 (1983) 589;

[12] M. Dubois-Violette, M. Talon and C.M. Viallet, *Comm. Math. Phys.* 102 (1985) 105;
Phys. Lett. B158 (1985) 231;
Ann. Inst. Henri Poincaré 44 (1986) 103;
M. Dubois-Violette, M. Henneaux, M. Talon and C.M. Viallet, *Phys. Lett.* B267 (1991) 81;
M. Henneaux, *Comm. Math. Phys.* 140 (1991) 1;

[13] S. P. Sorella, *Algebraic characterization of the Wess-Zumino consistency conditions in gauge theories*, UGVA-DPT 1992/08-781, to appear in Comm. Math. Phys.;

[14] F. Delduc, F. Gieres and S. P. Sorella, *Phys. Lett.* B225 (1989) 367;
F. Delduc, C. Lucchesi, O. Piguet and S. P. Sorella, *Nucl. Phys.* B346 (1990) 313;
E. Guadagnini, N. Maggiore and S. P. Sorella, *Phys. Lett.* B255 (1991) 65;
C. Lucchesi, O. Piguet and S. P. Sorella, *Renormalization and finiteness of topological BF theories* , MPI-ph/92-57, UGVA-DPT-92/07/773; to appear in Nucl. Phys. B;

[15] D. Birmingham and M. Rakowski, *Phys. Lett.* B269 (1991) 103;
Phys. Lett. B272 (1991) 217;
Phys. Lett. B275 (1992) 289;
Phys. Lett. B289 (1992) 271;

[16] G. Bandelloni, *Nuovo Cim.* 88A (1985) 1;
G. Bandelloni, *Nuovo Cim.* 88A (1985) 31;

[17] E. Kraus and K. Sibold, *Nucl. Phys.* B372 (1992) 113;
E. Kraus and K. Sibold, *Ann. Phys. (N.Y.)* 219 (1992) 349;

[18] G. Bandelloni, C. Becchi, A. Blasi and R. Collina, *Nucl. Phys.* B197 (19347) 1982;

[19] F. Bastianelli and P. Van Niewenhuizen, *Trace anomalies and Quantum Mechanics*, USITP-92-02, CERN-TH.6512/92;

[20] S. Deser and A. Schwimmer, *Geometric Classification of Conformal Anomalies in Arbitrary Dimensions* , Brandeis BRX-343, SISSA 14/93/EP;

[21] M. Dubois-Violette, M. Henneaux, M. Talon and C.M. Viallet, *Phys. Lett.* B289 (1992) 361;

[22] J.A. Dixon, *Cohomology and Renormalization of gauge theories I, II, III*,
Imperial College preprint-1977;

[23] L. Bonora, M. Bregola and L. Lucaroni, *Anomalies and cohomology*, Lectures given at the Ferrara University, SISSA 126/ep (dec. 89),(*see in particular Appendix A*);
O' Raifeartaigh, *Group structure of gauge theories*, Cambridge University press, 1986;

[24] F. Langouche, *Phys. Lett.* B148 (1984) 93;