

# On non-perturbative extensions of anti-de-Sitter algebras

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ABSTRACT: Motivated by the study of branes in curved backgrounds, we investigate the construction of non-perturbative extensions of the super-isometry algebra  $\text{osp}^*(8|4)$  of the  $\text{AdS}_7 \times S^4$  background of M-theory. This algebra is not a subalgebra of  $\text{osp}(1|32)$  and its non-perturbative extension can therefore not be obtained by embedding in this simple superalgebra. We show how, instead, it is possible to construct an extension directly by solving the Jacobi identities. This requires, in addition to the expected non-perturbative charges, the introduction of new charges which appear in the  $\{Q, Q\}$  bracket only via a linear combination with the bosonic generators of the isometry algebra. The resulting extended algebra has the correct flat-space limit, but it is not simple and the non-perturbative charges do not commute with the super-isometry generators. We comment on the consequences of this structure for the representation theory and on possible alternatives to our construction.

KEYWORDS: M-theory, superalgebras, D-branes.

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## 1. Introduction

Non-perturbative charges in supersymmetry algebras are very important for our understanding of M-theory, since they are related to the presence of branes and the construction of BPS bounds for these states, which guarantee their stability. Whereas the central extension of the flat-space super-Poincaré algebra has been understood for a long time and used in many applications, a similar extension for the super-isometry algebras of the other supersymmetric vacua has not been used so far and, as a matter of fact, has not yet been constructed explicitly.<sup>1</sup>

There is a widely expressed expectation that the simple algebra  $\text{osp}(1|32)$  plays the role of a “universal” algebra for M-theory, in the sense that the non-perturbatively extended isometry algebras of the supersymmetric vacua can be obtained as subalgebras. To be more precise, it is known that the maximal central extension of the super-translation algebra of flat space can be obtained as a contraction of the  $\text{osp}(1|32)$  algebra, but the Lorentz generators have to be added by hand.<sup>2</sup> This construction however, uses the semi-direct sum structure present in the flat-space algebra: the bracket of two supertranslation generators never produces a Lorentz generator. This structure is not present in the isometry algebras of the other vacua, where the  $\{Q, Q\}$  bracket closes into all bosonic generators. A quick glance at the literature shows that indeed, problems arise. For instance, the isometry superalgebra

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<sup>1</sup>We prefer to refer to these additional charges in the algebra as “non-perturbative” as opposed to “central” in order to avoid confusion: the charges in this letter will be far from central.

<sup>2</sup>The superalgebra “expansion” procedure of Azcárraga *et al.* [1] provides an alternative way to incorporate the Lorentz generators.

of the  $\text{AdS}_7 \times S^4$  vacuum, which is usually denoted as  $\text{osp}^*(8|4)$ , is *not* contained in  $\text{osp}(1|32)$ .<sup>3</sup>

There is a large body of work based on the  $\text{osp}^*(8|4)$  algebra. For instance, it has been used to determine the supergravity Kaluza-Klein spectrum [4], it is a crucial ingredient in the construction of the world-volume action for the supermembrane [5] in the associated background and it plays an important role in checks of the conjectured eleven-dimensional version of the AdS/CFT correspondence [6]. This algebra is thus likely to be present, in some form or another, in the full algebra that incorporates perturbative as well as non-perturbative physics. Given the comments in the previous paragraph, one may be tempted to conjecture that  $\text{osp}^*(8|4)$  is in fact all there is, and that it already contains the information about non-perturbative states [7]. This is, in our opinion, unlikely to be correct. One reason is that such a structure is not compatible with a flat space-time limit, in which central charges are known to appear. Moreover, it is known that the space-time superalgebra of the supermembrane in the  $\text{AdS}_7 \times S^4$  background has to contain, apart from the  $\text{osp}^*(8|4)$  generators, non-perturbative charges [8] corresponding to winding or infinitely extending membranes. This suggests that a larger algebra can indeed be constructed.

In the present letter we will consider extensions of  $\text{osp}^*(8|4)$  obtained by adding new bosonic generators yet keeping the number of fermionic generators fixed (less conservative options are possible and will be discussed elsewhere; see also the discussion section). Moreover, we will keep the commutators of the existing bosonic generators fixed. This restriction is motivated by the fact that, as we have argued above, the isometry algebra is reproduced by supermembrane charges even when central charges are taken into account. Because the anti-commutators of the fermions will get modified, the original algebra cannot be a sub-superalgebra of the extended one. When extending the  $\{Q, Q\}$  bracket we will therefore be guided by the requirement that the BPS equations of the old algebra should be obtainable from the new bracket by setting all new generators to zero.<sup>4</sup> This corresponds to the condition that the extended algebra is also valid in the free theory, *i.e.* that no phase transition and associated change in the multiplet structure occurs at zero coupling (a reasonable assumption, since such non-smooth behaviour at zero coupling would invalidate perturbation theory altogether).

In the first part of this letter we then show that it is indeed possible to construct a non-perturbative extension of the  $\text{osp}^*(8|4)$  algebra along these lines, by starting from a very general Ansatz and solving the Jacobi identities. A fact which one observes at a rather early stage in this construction is that additional bosonic charges are required, charges which are not present in the  $\text{osp}(1|32)$  algebra. These appear in the  $\{Q, Q\}$  bracket, but only in linear combination with the bosonic generators of the non-extended isometry algebra.

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<sup>3</sup>One of the reasons why this subtlety is often not observed is that the *bosonic* isometry algebra of the  $\text{AdS}_7 \times S^4$  background does occur as a subalgebra of  $\text{osp}(1|32)$ ; see Bars *et al.* [2] and Bandos *et al.* [3] for more on this issue. This fact will actually be important for us later (section 2.4).

<sup>4</sup>This condition is violated if one tries to use the  $\text{osp}(1|32)$  algebra as a starting point for a non-perturbative extension, since in this algebra the  $\text{so}(6, 2) \times \text{so}(5)$  isometry generators appear in the  $\{Q, Q\}$  bracket with coefficients which are different from the ones in  $\text{osp}^*(8|4)$ . While such an approach has been attempted in the literature, we regard the BPS relations as essential and will therefore not follow this route.

Without these additional charges it is impossible to satisfy the Jacobi identities.<sup>5</sup> As a result, the maximally extended superalgebra we find is *not* simple, but instead contains an ideal which is isomorphic to the *bosonic* isometry algebra of the background. Its structure is depicted in the diagram below:

$$\begin{array}{ccc}
 \text{osp}(1|32) & \oplus & (\text{so}(6, 2) \oplus \text{so}(5)) \\
 \{Q, \hat{M}^{(8,5)}, Z^{(2|1)}, Z^{(4|2)}\} & & \{\hat{W}^{(8,5)}\} \\
 \searrow & & \swarrow \\
 & \text{osp}^*(8|4) & \\
 & \{Q, M^{(8,5)} = \hat{M}^{(8,5)} + \hat{W}^{(8,5)}\} & 
 \end{array} \tag{1.1}$$

It is important to note that the isometry algebra of the background does not correspond to the second term in the sum, but instead is diagonally embedded. By taking the second term to be isomorphic to the bosonic isometry algebra of flat space-time, we show that the same “diagonalisation” procedure leads to the centrally extended super-Poincaré algebra. This provides evidence for a complete M-theory algebra of the form  $\text{osp}(1|32) \oplus G$  where  $G$  contains at least the bosonic isometry algebras of the M-theory vacua as subalgebras (the specific option of  $\text{osp}(1|32) \oplus \text{osp}(1|32)$  has been suggested in the literature, though based on different arguments, see *e.g.* [15]).

The algebra obtained in this way should satisfy several physical requirements. First of all, there should exist a contraction to the flat, maximally extended super-Poincaré algebra (broken to  $\text{iso}(6, 1) \oplus \text{so}(4)$ ). We show that our algebra indeed satisfies this important requirement.

Secondly, we should argue that the representation theory of the extended algebra contains multiplets which correspond to the multiplets of the original algebra. Despite the fact that the non-extended algebra cannot be a sub-superalgebra of the extended one (in which case this proof would be trivial), its structure is hidden in the larger algebra. This in principle makes it possible that the relevant physical information present in the non-extended algebra (like the content of the supergraviton and Kaluza-Klein multiplets and the BPS relations for them) can be “lifted” to the extended algebra. One simple possibility, which is realised in the case of the super-Poincaré extension, is that there exist representations in which all new generators act trivially on all states. However, it turns out this is not case for our algebra. This is essentially due to the fact that the commutation relations between the supercharges and the new bosonic generators are *non-trivial* this is why calling these new charges “central” would be even less correct than in flat space-time, and we instead prefer to call these charges “non-perturbative”).

Another option could be that, despite the fact that the action of the new charges is non-trivial on the original  $\text{osp}^*(8|4)$  multiplets, this action is such that one still preserves the old BPS relations for these states. This can happen if the expectation values of the

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<sup>5</sup>If one does not add such charges, one is extremely limited in the possible non-perturbative extensions. This has been analysed some time ago in the context of  $N = 2, 3$  and 4 superconformal algebras by Bedding [9, 10] and appeared more recently in [11]; see also [12, 13, 14]. For instance, in case of the  $\text{AdS}_5 \times S^5$  algebra the result is that only a “trivial”  $\mathfrak{u}(1)$  extension is allowed by the Jacobi identities [11]. However, we consider such an algebra without any other charges undesirable for reasons mentioned above.

new generators on these states vanish, or conspire and cancel out from the BPS equations. Because the new charges do not commute with the other generators, one would probably end up with multiplets with more states than in the non-extended multiplet, but this is not necessarily a disastrous problem. In principle these states could correspond to (bound) multi-particle states of the perturbative spectrum, generated by application of the new “non-perturbative” generators on the old perturbative states.<sup>6</sup> In section 2.4 we investigate this possibility, and show that indeed some states in the supergraviton multiplet might fulfill these requirements. However, there seem to be no general arguments to prove the existence of the lift, and in fact it seems unlikely that it exists (a definite proof would involve the construction of explicit representations of the full algebra).

We thus conclude that an extension of the super-isometry algebras of M-theory vacua is possible, but that a physically satisfactory construction will most likely involve the introduction of yet more generators, over and above the minimal ones that we have considered. We conclude this letter with a few comments on such other alternatives, which are currently under investigation.

## 2. M-theory on $\text{AdS}_7 \times S^4$ and non-perturbative charges

### 2.1 Constructing an extended $\text{osp}^*(8|4)$ algebra

The bosonic isometry algebra of the  $\text{AdS}_7 \times S^4$  background of M-theory is given by  $\text{so}(6, 2) \times \text{so}(5)$ . Its super-extension is the superalgebra  $\text{osp}^*(8|4)$  (in order to denote Lie superalgebras we will follow the notation used by Van Proeyen [16], see also Frappat *et al.* [17]). This superalgebra can be written conveniently using a 13-dimensional notation. With this convention the algebra has the commutation relations<sup>7</sup>

$$\begin{aligned} [M_{AB}^{(8)}, M_{CD}^{(8)}] &= \delta_{AC} M_{BD}^{(8)}, & [M_{IJ}^{(5)}, M_{KL}^{(5)}] &= \delta_{IK} M_{JL}^{(5)}, \\ [M_{AB}^{(8)}, Q] &= \frac{1}{8} (\Gamma_{AB} Q), & [M_{IJ}^{(5)}, Q] &= \frac{1}{8} (\Gamma_{IJ} Q) \end{aligned} \quad (2.1)$$

together with

$$\{Q, Q\} = (\Gamma^{AB} C_{(13)}^{-1}) M_{AB}^{(8)} - 2 (\Gamma^{IJ} C_{(13)}^{-1}) M_{IJ}^{(5)}. \quad (2.2)$$

Anti-symmetrisation on the appropriate indices on the right-hand side is understood.

As we have mentioned in the introduction, the superalgebra given above has been used extensively, and appears for instance in the space-time supersymmetry algebra of the supermembrane. Keeping track of total derivative terms, central charge terms have been observed to arise in the  $\{Q, Q\}$  bracket of the superalgebra of the membrane in  $\text{AdS}_7 \times S^4$ ; see the work of Sato [18, 19] and Furuuchi *et al.* [8]. Just as in flat space one encounters

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<sup>6</sup>These new states can not, however, correspond to branes, since we expect that branes decouple from the spectrum in the zero coupling limit. The perturbative multi-particle states, on the other hand, should be still present in this limit.

<sup>7</sup>The  $A, B, \dots$  indices are  $\text{so}(6, 2)$  indices and  $I, J, \dots$  are  $\text{so}(5)$  ones. The eight-dimensional spinors are anti-Weyl. Note that we take the gamma matrices of these two algebras to be commuting; this makes the relation to the eleven-dimensional algebra more complicated but simplifies most of the other calculations in this paper.

first of all a two-form charge. One does not see a five-form charge in the membrane algebra since the only term one can write down vanishes identically, but this charge is expected to appear in the matrix model (such charges have indeed been found in the pp-wave limit, see Sugiyama and Yoshida [20] for a membrane analysis and Hyun and Shin [21] for a matrix model computation).<sup>8</sup> Motivated by these facts, we thus set out to construct a maximal non-perturbative extension of (2.2) and the other commutators.

The supercharges appearing in (2.2) are in the 32-dimensional representation of the bosonic subalgebra, or more precisely, in the  $(\mathbf{8}_s, \mathbf{4})$ . In order to determine the maximal extension of the  $\{Q, Q\}$  bracket, one has to consider the symmetric tensor product

$$\left( (\mathbf{8}_s, \mathbf{4}) \otimes (\mathbf{8}_s, \mathbf{4}) \right)_{\text{symm}} = (\mathbf{28}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{10}) \oplus (\mathbf{28}, \mathbf{5}) \oplus (\mathbf{35}^-, \mathbf{10}). \quad (2.3)$$

From the representation theory point-of-view, the *minimal* form of the maximal extension of the algebra, compatible with the presence of an  $\text{osp}^*(8|4)$  “core”, should thus at the very least have a  $\{Q, Q\}$  commutator of the form

$$\begin{aligned} \{Q, Q\} = & (\Gamma^{AB} \mathcal{C}_{(13)}^{-1}) M_{AB}^{(8)} - 2 (\Gamma^{IJ} \mathcal{C}_{(13)}^{-1}) M_{IJ}^{(5)} \\ & + c (\Gamma^{AB} \Gamma^I \mathcal{C}_{(13)}^{-1}) Z_{ABI}^{(2|1)} + d (\Gamma^{ABCD} \Gamma^{IJ} \mathcal{C}_{(13)}^{-1}) Z_{ABCDIJ}^{(4|2)-}. \end{aligned} \quad (2.4)$$

The minus sign on the charge  $Z^{(4|2)-}$  denotes that we have chosen this charge to be anti-self-dual. Both the left-hand side and the right-hand side contain 528 physical components. We should stress here that (2.4) does not appear in the commutation relations of the simple algebra  $\text{osp}(1|32)$ : with the normalisation of the isometry generators chosen as in (2.1), the relative coefficient between the  $M^{(8)}$  and  $M^{(5)}$  generators in the  $\{Q, Q\}$  bracket of this algebra comes out as 1, as opposed to the  $-2$  in (2.4).

Apart from the fact that the algebra based on the extension (2.4) eventually turns out to be inconsistent with the Jacobi identities for any non-zero values of  $c$  and  $d$ , there is also a simpler, counting argument that suggests that this Ansatz cannot be correct. Namely, upon contracting the generators that appear in the expressions above to the flat space generators, one knows that the isometry generators  $M$  of flat space-time disappear from the  $\{Q, Q\}$  bracket, leaving only the translation generators. Therefore, this contraction of (2.4) will necessarily lead to a disbalance between the number of components on the left-hand side and the right-hand side, which one knows does not happen in the maximally extended super-Poincaré algebra.

We therefore introduce additional charges  $W_{AB}^{(8)}$  and  $W_{IJ}^{(5)}$  and change the bracket of supersymmetry charges (2.4) to

$$\begin{aligned} \{Q, Q\} = & (\Gamma^{AB} \mathcal{C}_{(13)}^{-1}) \left( M_{AB}^{(8)} + \tilde{a} W_{AB}^{(8)} \right) + (\Gamma^{IJ} \mathcal{C}_{(13)}^{-1}) \left( -2 M_{IJ}^{(5)} + \tilde{b} W_{IJ}^{(5)} \right) \\ & + c (\Gamma^{AB} \Gamma^I \mathcal{C}_{(13)}^{-1}) Z_{ABI}^{(2|1)} + d (\Gamma^{ABCD} \Gamma^{IJ} \mathcal{C}_{(13)}^{-1}) Z_{ABCDIJ}^{(4|2)-}, \end{aligned} \quad (2.5)$$

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<sup>8</sup>One should not confuse these superalgebras with the world-volume algebras considered by Bergshoeff *et al.* [22]; the algebras considered in that paper involve world-volume spinors and *are* maximally extended.

We also make the most general Ansatz for the other brackets which is compatible with representation theory (these can be found in equations (A.6)–(A.11) in the appendix). In order to fix the coefficients, we have systematically analysed the Jacobi identities, details of which can be found in the appendix.

The upshot of this analysis is the following. Firstly, one observes that one actually does not have to go through all of the Jacobi identities in order to fix the coefficients. Once the structure of the algebra involving the  $\{M^{(8)}, W^{(8)}, M^{(5)}, W^{(5)}\}$  generators is fixed, one can diagonalise this sector. One then finds that an ideal isomorphic to  $\mathfrak{so}(6, 2) \oplus \mathfrak{so}(5)$  is generated by the linear combinations

$$\hat{W}^{(8)} := \frac{1}{2\tilde{m} - 1} \left( \tilde{m} M^{(8)} - W^{(8)} \right) \quad \text{and} \quad \hat{W}^{(5)} := \frac{1}{2\tilde{n} - 1} \left( \tilde{n} M^{(5)} - W^{(5)} \right) \quad (2.6)$$

The parameters  $\tilde{m}$  and  $\tilde{n}$  are related (see (A.18)) and correspond to a single rescaling freedom in the algebra. Note that this ideal is isomorphic but not identical to the bosonic isometry algebra of the background. The only non-trivial commutators involving the  $\hat{W}$  charges are in fact (suppressing indices for brevity)

$$[\hat{W}^{(8)}, \hat{W}^{(8)}] = \hat{W}^{(8)}, \quad [\hat{W}^{(5)}, \hat{W}^{(5)}] = \hat{W}^{(5)}, \quad (2.7)$$

while other ones vanish (and the  $\hat{W}$  charges in particular map the supercharges to zero). The other linear combination that one can make,

$$\hat{M}^{(8)} := \frac{1}{2\tilde{m} - 1} \left( (\tilde{m} - 1) M^{(8)} + W^{(8)} \right) \quad \text{and} \quad \hat{M}^{(5)} := \frac{1}{2\tilde{n} - 1} \left( (\tilde{n} - 1) M^{(5)} + W^{(5)} \right) \quad (2.8)$$

combines with  $Q$ ,  $Z^{(2|1)}$  and  $Z^{(4|2)}$ . By computing the  $\{Q, Q\}$  bracket one sees that the  $\hat{M}^{(8)}$  and  $\hat{M}^{(5)}$  generators now appear with relative coefficient one. In fact, these charges turn out to generate the algebra  $\mathfrak{osp}(1|32)$ . In terms of the original, unhatted generators, the algebra is given by (2.1), the supercharge bracket (2.5) (with fixed values of  $c$  and  $d$  in terms of  $\tilde{m}$ ), as well as the commutation relations

$$[M^{(8)}, W^{(8)}] = W^{(8)}, \quad [W^{(8)}, W^{(8)}] = \tilde{m}(\tilde{m} - 1)M^{(8)} + W^{(8)} \quad (2.9)$$

$$[M^{(5)}, W^{(5)}] = W^{(5)}, \quad [W^{(5)}, W^{(5)}] = \tilde{n}(\tilde{n} - 1)M^{(5)} + W^{(5)}. \quad (2.10)$$

Our non-perturbative charges are even less “central” than in the super-Poincaré case; the brackets with the supercharges mimic the brackets in (2.1):

$$\begin{aligned} [W^{(8)}, Q] &= \frac{\tilde{m}}{8} \Gamma Q, & [W^{(5)}, Q] &= \frac{\tilde{n}}{8} \Gamma Q, \\ [Z^{(2|1)}, Q] &= C \Gamma Q, & [Z^{(4|2)}, Q] &= D \Gamma Q, \end{aligned} \quad (2.11)$$

(where  $C$  and  $D$  are again determined in terms of  $\tilde{m}$ ). The appearance of non-trivial commutators like (2.11) is in fact very general. They appear whenever the non-perturbative charges transform non-trivially under the action of the isometry generators, and these isometry generators also appear on the right-hand side of the  $\{Q, Q\}$  bracket. In this case a non-trivial  $[Q, Z]$  commutator is required in order to make the  $(Q, Q, Z)$  Jacobi identity work.

We also find that the new charges do not commute among each other:

$$\begin{aligned}
[M^{(8)}, Z^{(2|1)}] &= \tilde{m}^{-1} [W^{(8)}, Z^{(2|1)}] = Z^{(2|1)}, \\
[M^{(5)}, Z^{(2|1)}] &= \tilde{n}^{-1} [W^{(5)}, Z^{(2|1)}] = \frac{1}{2} Z^{(2|1)}, \\
[M^{(8)}, Z^{(4|2)}] &= \tilde{m}^{-1} [W^{(8)}, Z^{(4|2)}] = 2 Z^{(4|2)}, \\
[M^{(5)}, Z^{(4|2)}] &= \tilde{n}^{-1} [W^{(5)}, Z^{(4|2)}] = Z^{(4|2)}.
\end{aligned} \tag{2.12}$$

The commutators between  $Z^{(2|1)}$  and  $Z^{(4|2)}$  are similarly non-trivial but more complicated, see the appendix for details. By a rescaling of the fermions one can furthermore eliminate the dependence on  $\tilde{m}$  and  $\tilde{n}$ .

To summarise, the expressions (2.5)–(2.12) define the maximally extended superisometry algebra of the  $\text{AdS}_7 \times S^4$  vacuum of M-theory. This algebra has the structure

$$\text{osp}(1|32) \oplus (\text{so}(6, 2) \oplus \text{so}(5)), \tag{2.13}$$

but it is very important that the non-extended algebra  $\text{osp}^*(8|4)$  is embedded in this algebra in a non-trivial way, as depicted in (1.1) given in the introduction. We should stress that, while it may make sense to write the algebra in a basis which makes (2.13) manifest (*i.e.* using the hatted generators), this is not the natural basis one would encounter in for instance the supermembrane algebra. Moreover, it obscures the physical meaning of the  $M$  generators as isometry generators of the background.

Note that the Jacobi identities leave *no room* for the inclusion of a parameter which can smoothly tune the new charges to zero. However, there is some freedom to completely discard the  $Z^{(4|2)}$  charge, which may be important for certain world-volume symmetry algebras.

## 2.2 The flat space-time limit

If our algebra is to be the correct one, it has to pass several physical consistency checks. One check is that there is a flat space-time limit in which the algebra reduces to the centrally extended, super-iso(6, 1)  $\times$  so(4) algebra. (Obviously, one can never recover the eleven-dimensional super-Poincaré vacuum, since the local Lorentz algebra in the  $\text{AdS}_7 \times S^4$  space-time is broken from so(10, 1) to so(6, 1)  $\oplus$  so(4), and contraction does not change the number of generators).

To take the flat space-time limit of the  $\text{AdS}_7 \times S^4$  algebra presented in the previous section we first rewrite the algebra in a manifestly so(6, 1)  $\oplus$  so(4) symmetric form. The isometry generators in the two formulations are related according to

$$M_{\underline{1}a}^{(8)} \rightarrow RP_a, \quad M_{ab}^{(8)} \rightarrow M_{ab}^{(8)}, \quad W_{\underline{1}a}^{(8)} \rightarrow R^\Sigma W_a^{(1)}, \quad W_{ab}^{(8)} \rightarrow RZ_{ab}^{(2)} \tag{2.14}$$

$$M_{5i}^{(5)} \rightarrow RP_i, \quad M_{ij}^{(5)} \rightarrow M_{ij}^{(5)}, \quad W_{5i}^{(5)} \rightarrow R^\Sigma W_i^{(1)}, \quad W_{ij}^{(5)} \rightarrow RZ_{ij}^{(2)}. \tag{2.15}$$

where we have split the indices as  $A = (\underline{1}, a)$  and  $I = (i, 5)$ . Here  $R$  is the dimensionless radius of the  $\text{AdS}_7$  factor (*i.e.* the dimensionful radius divided by the Planck length) while



the radius of the  $S^4$  equals  $R/2$ . We will comment below on the value of the parameter  $\Sigma$  when we take the limit to flat space-time. The  $Z^{(2|1)}$  generators split as

$$Z_{\underline{1}a|i}^{(2|1)} \rightarrow RZ_{(1,1)}^{(2)}, \quad Z_{\underline{1}a|5}^{(2|1)} \rightarrow RZ_{(1,4)}^{(5)}, \quad Z_{ab|5}^{(2|1)} \rightarrow RZ_{(2,0)}^{(5)} \equiv RZ_{(5,0)}^{(5)}, \quad Z_{ab|i}^{(2|1)} \rightarrow RZ_{(2,3)}^{(5)}, \quad (2.16)$$

while the  $Z^{(4|2)}$  generators split and scale as

$$Z_{\underline{1}abc|ij}^{(4|2)} \rightarrow RZ_{(3,2)}^{(5)}, \quad Z_{\underline{1}abc|i5}^{(4|2)} \rightarrow RZ_{(4,1)}^{(5)}. \quad (2.17)$$

We used the notation  $Z_{(p,q)}^{(2)}$  and  $Z_{(p,q)}^{(5)}$  to denote a two-form resp. five-form which transforms as a  $(p, q)$ -form under the  $\mathfrak{so}(6, 1) \oplus \mathfrak{so}(4)$  algebra. Counting the generators, we are left with precisely the number of central charges expected in the flat space-time algebra, except for the two additional  $W^{(1)}$  charges. However, as we will see, these additional charges do not pose problems. Finally, the supercharges are redefined as

$$Q \rightarrow \sqrt{R}Q. \quad (2.18)$$

The flat space-time limit is now obtained by taking the  $R \rightarrow \infty$  limit. In order to compare the result with a direct construction of the algebra in flat space-time, one has to express the  $\mathfrak{so}(6, 2) \oplus \mathfrak{so}(5)$  gamma matrices in terms of  $\mathfrak{so}(6, 1) \oplus \mathfrak{so}(4)$  ones (this can be done by using  $\mathfrak{so}(10, 1)$  intermediate notation as explained in (A.3) and (A.4) in the appendix). One easily finds agreement between these two results. The  $\{Q, Q\}$  bracket turns into

$$\begin{aligned} \{Q, Q\} = & (\tilde{\Gamma}^a C^{-1})P_a + (\tilde{\Gamma}^i C^{-1})P_i + (\tilde{\Gamma}^{ab} C^{-1})Z_{ab}^{(2)} + (\tilde{\Gamma}^{ij} C^{-1})Z_{ij}^{(2)} + (\tilde{\Gamma}^{ai} \tilde{\Gamma} C^{-1})Z_{ai}^{(2)} \\ & + (\tilde{\Gamma}^{abijk} C^{-1})Z_{abijk}^{(5)} + (\tilde{\Gamma}^{aijkl} C^{-1})Z_{aijkl}^{(5)} + (\tilde{\Gamma}^{abcde} \tilde{\Gamma} C^{-1})Z_{abcde}^{(5)} \\ & + (\tilde{\Gamma}^{abcdi} C^{-1})Z_{abcdi}^{(5)} + (\tilde{\Gamma}^{abcij} C^{-1})Z_{abcij}^{(5)}, \end{aligned} \quad (2.19)$$

where  $\tilde{\Gamma}^a$  and  $\tilde{\Gamma}^i$  are gamma matrices in  $C(6, 1)$  and  $C(4, 0)$  respectively,  $\tilde{\Gamma}$  denotes the chirality matrix in  $C(4, 0)$  and the charge conjugation matrix is given by  $C \equiv C_{(6,1)} \times C_{(4)}$ . Note that the four-dimensional spinors are not Weyl, which explains the appearance of  $\tilde{\Gamma}$ , yet dualisation has been freely used in the seven-dimensional sector (for instance to identify  $Z_{(2,0)}^{(5)} \equiv Z_{(5,0)}^{(5)}$  in (2.16)). The bracket (2.19) is precisely what one would get if one were to construct this flat-space central extension from scratch.

Several comments are in order here. Firstly, note that flat space five-form  $Z^{(5)}$  arises both from the  $Z^{(2|1)}$  and the  $Z^{(2|4)}$  charges. Secondly, the parameter  $\Sigma$  in (2.14) with which we scale the  $W^{(8)}$  and  $W^5$  charges should lie in the range

$$\frac{1}{2} < \Sigma \leq 1. \quad (2.20)$$

For  $\frac{1}{2} < \Sigma < 1$ , the charges  $W_a^{(1)}$  and  $W_i^{(1)}$  decouple from the remainder of the algebra in the flat space-time limit: they disappear from the  $\{Q, Q\}$  bracket and become mutually commuting, while keeping their transformation property under the action of the  $M$  generators.<sup>9</sup> In the case  $\Sigma = 1$ , these two one-form charges combine with the momenta, and

<sup>9</sup>It might be that this one-form charge is related to the possibility of constructing kappa-symmetric open membranes which end on a string [23].

appear only through the combinations  $(P_a + W_a)$  and  $(P_i + W_i)$  in the full algebra. The other linear combination has trivial commutators with all other generators, including the Lorentz generators  $M$ . This case is perhaps more natural and resembles a similar situation in string theory, where winding charges combine with momenta [24].

Finally, note that it was crucial for the correct flat space limit that the generators  $M^{8,5}$  and  $W^{8,5}$  get scaled away from the  $\{Q, Q\}$  commutator, but *not*  $\hat{M}^{(8,5)}$ ,  $\hat{W}^{(8,5)}$ . The latter option would lead to a reduction of the number of generators on the right hand side of this commutator, while the left hand side would still contain the same number of components. Hence, to take the the correct flat space limit it is crucial to correctly identify the bosonic isometry generator as belonging to the *diagonal*  $so(6, 2) \times so(4)$  within the algebra (2.13).

### 2.3 The super-Poincaré algebra from a diagonal embedding

While the presence of  $osp(1|32)$  in our algebra is reminiscent of the structure of the maximally extended super-Poincaré algebra, the isometry generators are embedded in a different way, namely through “diagonal embedding” instead of through a semi-direct sum. In the present section we would like to comment on this difference and show how the diagonal embedding approach also works for the flat space-time situation.

Remember that a contraction of the  $osp(1|32)$  algebra leads to the maximally extended eleven-dimensional super-*translation* algebra [25], spanned by the translation generators  $P$  and two- and five-form central charges  $Z^{(2)}$  and  $Z^{(5)}$ . To obtain the full super-Poincaré algebra, one has to add the Lorentz generators of the  $so(10, 1)$  algebra separately: one starts with the semi-direct sum  $so(10, 1) \in osp(1|32)$  (where  $so(10, 1)$  acts non-trivially on  $osp(1|32)$ ), and does not scale these generators in the process of contraction (see however [1]).

Alternatively, inspired by the structure that has appeared in the previous section, one could start from the algebra  $osp(1|32) \oplus so(10, 1)$  (where “ $\oplus$ ” denotes the *direct* sum). Let us denote the generators of  $so(10, 1)$  in  $osp(1|32)$  with  $\hat{Z}^{(2)}$  and the ones sitting in the other  $so(10, 1)$  with  $\hat{M}^{(2)}$ . Decomposing the  $\{Q, Q\}$  commutator with respect to the  $so(10, 1)$  subalgebra of  $osp(1|32)$ , one gets the familiar expression

$$\{Q, Q\} \sim \hat{P} + \hat{Z}^{(2)} + \hat{Z}^{(5)}. \quad (2.21)$$

The rest of the  $osp(1|32)$  commutators can be written schematically as

$$[\hat{Z}^{(2)}, \hat{Z}^{(2)}] \sim \hat{Z}^{(2)} \quad [\hat{Z}^{(2)}, \hat{Z}^{(5)}] \sim \hat{Z}^{(2)} + \hat{Z}^{(5)} \quad [\hat{Z}^{(5)}, \hat{Z}^{(5)}] \sim \hat{Z}^{(2)} + \hat{Z}^{(5)} \quad (2.22)$$

$$[\hat{P}, \hat{Z}^{(2)}] \sim \hat{P} \quad [\hat{P}, \hat{Z}^{(5)}] \sim \hat{Z}^{(5)} \quad [\hat{P}, \hat{P}] \sim \hat{P} + \hat{Z}^{(2)}. \quad (2.23)$$

As in the AdS case, one identifies the Lorentz generators of the flat vacuum with a diagonal subalgebra of the two  $so(10, 1)$  algebras,

$$M^{(2)} \equiv \hat{M}^{(2)} + \hat{Z}^{(2)}, \quad Z^{(2)} \equiv \hat{M}^{(2)} - \hat{Z}^{(2)}, \quad P \equiv \hat{P}. \quad (2.24)$$

In taking the contraction to the flat space limit, one scales the generators as

$$Q \rightarrow \sqrt{R}Q, \quad P \rightarrow RP, \quad M \rightarrow M, \quad Z^{(2)} \rightarrow RZ^{(2)}, \quad Z^{(5)} \rightarrow RZ^{(5)}. \quad (2.25)$$

It is now obvious that by rewriting (2.22) in terms of the new variables  $M^{(2)}$  and  $Z^{(2)}$  and applying the scaling (2.25), one arrives at the commutation relations of the full super-Poincaré algebra (with trivial commutation relations between all central charges and a nontrivial action of the Lorentz generators  $M^{(2)}$  on all central charges). Note that before the contraction, the  $\{Q, Q\}$  commutator contains, just as in the AdS case, a linear combination of the bosonic generators and central charges. Additionally, the bosonic generators drop out from the commutator only after having taken the  $R \rightarrow \infty$  limit.

From the two explicit examples we have worked out in detail in this note, it is suggestive that the general form of the complete M-theory algebra could be of the form  $\text{osp}(1|32) \oplus G$ , where  $G$  is a minimal *bosonic* algebra that incorporates the isometry algebras of all M-theory vacua. However, we still have to analyse the representation theory of our algebra and compare it with the representation theory of  $\text{osp}^*(8|4)$ .

## 2.4 Representation theory and the graviton multiplet

If the non-perturbatively extended algebra presented in this paper is to be compatible with existing literature, we should show (at the very least) that the massless graviton multiplet of the non-extended  $\text{osp}^*(8|4)$  algebra “lifts” to a representation of the bigger algebra. The fact that the  $\text{osp}^*(8|4)$  algebra is not a sub-superalgebra of the algebra we have constructed means that, apart from the fact that we have introduced new generators (non-perturbative charges), we have also modified the action of the “old” generators (supercharges). A “lift” of existing supermultiplets of  $\text{osp}^*(8|4)$  can therefore not be just a simple embedding. Since the new generators of the extended algebra have non-trivial commutation relations with the supercharges, the various states in a supermultiplet will in principle transform non-trivially under the action of the new charges. Hence, a single state of the old algebra is generically lifted to a set of states with the same values of the old quantum numbers, yet with non-zero values of the new charges.

The graviton multiplet is a short multiplet: when constructing the multiplet by repeatedly acting on the lowest energy states with energy-raising supersymmetry operators, one encounters some states which have zero norm and are therefore absent from the multiplet. Consider now the eigenvalue problem

$$\det \left( \langle \phi | \{Q_\alpha, Q_\beta\} - \lambda \delta_{\alpha\beta} | \phi \rangle \right) = 0 \quad (2.26)$$

for a given state  $|\phi\rangle$ . Because of the existence of null vectors some supercharges act trivially on  $|\phi\rangle$ , and the polynomial on the left-hand side will have a number of zeroes at  $\lambda = 0$ . Some of these are trivial: for *e.g.* a ground state, where we know that all energy-lowering operators should vanish identically, we find a factor  $\lambda^{16}$ . The non-trivial factor(s) of this polynomial constitute the BPS equation(s) for the given state  $|\phi\rangle$ .

The unitary irreducible representations of  $\text{osp}^*(8|4)$  are uniquely labeled [4] by representations of its maximal compact bosonic subalgebra  $\text{so}(2) \times \text{su}(4) \times \text{so}(5)$ .<sup>10</sup> The BPS

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<sup>10</sup>Note that each unitary irrep of the compact subgroup  $\text{so}(2) \times \text{su}(4)$  gives rise to an *infinite-dimensional* representation of the full non-compact group  $\text{so}(6, 2)$ . This reflects the general fact that there are no finite-dimensional unitary representations of non-compact groups.

equations on a state are expressed in terms of the associated quantum numbers. By “lifted state” we now mean a state in a multiplet of the extended algebra which transforms in the same way under the bosonic generators inherited from the old algebra, and is annihilated by the same number of supercharges as before<sup>11</sup>. The BPS condition for the state in the extended algebra can be derived from

$$\langle \phi | \{Q, Q\} | \phi \rangle = \langle \phi | \left( M^{(8)} - 2M^{(5)} + W^{(8)} + W^{(5)} + Z^{(2|1)} + Z^{(4|2)} \right) | \phi \rangle. \quad (2.27)$$

The first two terms on the right-hand side are the same as in the  $\text{osp}^*(8|4)$  algebra, and hence vanish, when evaluated on any  $\text{osp}^*(8|4)$  BPS state. Using simple representation theory arguments one can show that the expectation values of *some* of the non-perturbative charges in *some* of the gravity multiplet states vanish. There are, for instance, several  $\text{AdS}_7$  scalars in the graviton multiplet [4] (the “dilaton”) which carry the  $\text{so}(2) \times \text{su}(4) \times \text{so}(5)$  quantum numbers  $[4, 0, 0, 0, 2, 0]$ . The tensor product with  $Z^{(2|1)}$  does not contain this dilaton representation, and the presence of  $\langle \phi | Z^{(2|1)} | \phi \rangle$  does therefore not modify the BPS equation for these particular states.

However, this simple argument cannot be used to deduce the absence of all non-perturbative charges from the BPS equation for all states. This is essentially due to the fact that there are charges which transform in the same way as either  $M^{(8)}$  or  $M^{(5)}$  under the action of the  $\text{so}(5)$  group. This then implies that their expectation values in the supergraviton states are generically non-vanishing. The only way they could drop out from the BPS equations is if they accidentally vanish for the given representation, or if they conspire together to give zero. It seems unlikely that this will happen in general, but a definite conclusion will require the explicit construction of the representations of the full algebra, which we will not attempt here.

### 3. Discussion

We have constructed a maximal non-perturbative extension of the super-isometry algebra of the  $\text{AdS}_7 \times S^4$  vacuum of M-theory by adding new bosonic charges to the isometry algebra  $\text{osp}^*(8|4)$  (a similar construction is possible for the other supersymmetric vacua of M-theory and those of string theory). These additional charges are even less central than those that appear in the extension of the super-Poincaré algebra: just as in the supersymmetric extension of conformal algebras [26] they commute neither with the other bosonic charges nor with the supercharges. While our algebra satisfies a couple of physical consistency checks, we have also argued that the structure might lead to a representation theory which does not produce the expected “lifted”  $\text{osp}^*(8|4)$  multiplets.

The extension constructed here exhibits additional non-abelian charges, which is a generic feature of extensions of AdS algebras. This implies that a direct interpretation of the new generators in terms of brane charges becomes more complicated than in flat

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<sup>11</sup>Lifting the representation of the diagonal subalgebra to the full one can be understood more easily by considering a toy example, for instance the lift of  $\text{su}(2)$  representations to  $\text{su}(2) \times \text{su}(2)$  in which it is embedded.

space-time. One particular situation in which the analysis may be simplified is in the Hpp-wave limit of the background discussed in this paper. Branes in this background have been studied using probe branes, boundary states and supergravity solutions, but a simple classification using representation theory of an extended algebra would be desirable; we intend to investigate this problem in the near future.

It is conceivable that our extension is still too conservative, and that the problems in finding a correct perturbative spectrum arise because an even larger algebra should be considered. Our analysis shows that one is then *forced* to introduce additional fermionic charges, which may perhaps lead to an  $\text{osp}(1|32) \oplus \text{osp}(1|32)$  structure. Fermionic extensions actually seem to be suggested by an analysis of the superalgebra of the matrix model in a pp-wave background, and we will report on this issue elsewhere [27].

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## A. Appendix

### A.1 Conventions and Clifford algebras

Our  $\text{so}(6, 2)$  spinors are taken to be anti-Weyl, so together with our conventions for the  $\text{so}(5)$  part we have

$$\Gamma^{A_1 \dots A_8} = -\epsilon^{A_1 \dots A_8}, \quad \Gamma^{I_1 \dots I_5} = \epsilon^{I_1 \dots I_5}. \quad (\text{A.1})$$

This is consistent with our choice for anti-self-duality of the charge  $Z^{(4|2)}$ ,

$$Z_{ABCD|IJ}^- = -\frac{1}{4!} \epsilon_{ABCD}{}^{EFGH} Z_{EFGH|IJ}^-. \quad (\text{A.2})$$

We use the notation  $C(t, s)$  for the Clifford algebra for a Lorentz-like metric with signature  $(t, s)$ . The explicit representation of  $C(6, 1) \otimes C(4, 0)$  by means of  $C(10, 1)$  is given by  $(a = 0, \dots, 6; i = 7, \dots, 10)$ ,

$$\begin{aligned} \tilde{\Gamma}^i &= \gamma^i, & \tilde{\Gamma}^a &= \gamma^* \gamma^a, \\ \gamma^* &= \gamma^{7 \dots 10}, & C &= \mathcal{C}_{(11)} \gamma^*, \end{aligned} \quad (\text{A.3})$$

while a representation of  $C(6, 2) \otimes C(5, 0)$  by means of  $C(10, 1)$  is given by

$$\begin{aligned} \Gamma^i &= \gamma^i \otimes 1, & \Gamma^y &= -i \gamma^* \otimes 1, \\ \Gamma^a &= \gamma^* \gamma^a \otimes \sigma^1, & \Gamma^{\underline{1}} &= -1 \otimes \sigma^2, \end{aligned} \quad (\text{A.4})$$

where, in the notation of footnote 7, the index ranges are  $(A = \underline{1}, 0, \dots, 6; I = 7, \dots, 10, y)$ . The charge conjugation matrix of this 13-dimensional representation and the Weyl projector in the  $\text{so}(6, 2)$  factor are given by

$$\mathcal{C}_{(13)} = \mathcal{C}_{(11)} \gamma^* \otimes 1, \quad \text{and} \quad \Gamma_{(9)} = 1 \otimes \sigma^3. \quad (\text{A.5})$$

### A.2 Solving the Jacobi identities

As explained in the main text, we have derived the non-perturbative extension of the  $\text{osp}^*(8|4)$  algebra by starting from a general Ansatz and solving the Jacobi identities. The key steps are presented below. The most important ingredient in our Ansatz is the bracket

$$\{Q, Q\} = aM^{(8)} + \tilde{a}W^{(8)} + bM^{(5)} + \tilde{b}W^{(5)} + cZ^{(2|1)} + dZ^{(4|2)}. \quad (\text{A.6})$$

where we have suppressed all indices and gamma matrices for brevity. We will use the standard  $\text{so}(6, 2)$  and  $\text{so}(5)$  brackets for the isometry generators. One may actually think of adding central charges to these  $[M, M]$  commutators, but apart from the reasons against this which were given in the introduction, one also quickly finds out that in this case it is impossible to satisfy the  $(Q, Q, Z)$  and  $(M, M, Z)$  Jacobi identities at the same time. The most general form of the commutators involving the generators  $M^{(8)}$ ,  $M^{(5)}$  and the new charges  $W^{(8)}$ ,  $W^{(5)}$  is

$$\begin{aligned} [M^{(8)}, W^{(8)}] &= \alpha M^{(8)} + \tilde{\alpha} W^{(8)}, & [W^{(8)}, W^{(8)}] &= \beta M^{(8)} + \tilde{\beta} W^{(8)} \\ [M^{(5)}, W^{(5)}] &= \gamma M^{(5)} + \tilde{\gamma} W^{(5)}, & [W^{(5)}, W^{(5)}] &= \delta M^{(5)} + \tilde{\delta} W^{(5)}. \end{aligned} \quad (\text{A.7})$$

These generators act on the fermionic charges as

$$\begin{aligned}
[M^{(8)}, Q] &= A \Gamma Q, & [M^{(5)}, Q] &= B \Gamma Q, \\
[W^{(8)}, Q] &= A_z \Gamma Q, & [W^{(5)}, Q] &= B_z \Gamma Q, \\
[Z^{(2|1)}, Q] &= C \Gamma Q, & [Z^{(4|2)}, Q] &= D \Gamma Q.
\end{aligned} \tag{A.8}$$

What remains is to fix the commutators with the  $Z^{(2|1)}$  and  $Z^{(4|2)}$  charges. These are parametrised as

$$\begin{aligned}
[M^{(8)}, Z^{(2|1)}] &= m Z^{(2|1)}, & [W^{(8)}, Z^{(2|1)}] &= \tilde{m} [M^{(8)}, Z^{(2|1)}], \\
[M^{(5)}, Z^{(2|1)}] &= n Z^{(2|1)}, & [W^{(5)}, Z^{(2|1)}] &= \tilde{n} [M^{(5)}, Z^{(2|1)}],
\end{aligned} \tag{A.9}$$

and

$$\begin{aligned}
[M^{(8)}, Z^{(4|2)}] &= \mu Z^{(4|2)}, & [W^{(8)}, Z^{(4|2)}] &= \tilde{\mu} [M^{(8)}, Z^{(4|2)}], \\
[M^{(5)}, Z^{(4|2)}] &= \nu Z^{(4|2)}, & [W^{(5)}, Z^{(4|2)}] &= \tilde{\nu} [M^{(5)}, Z^{(4|2)}].
\end{aligned} \tag{A.10}$$

The “standard” non-perturbative charges obey the commutators

$$\begin{aligned}
[Z^{(2|1)}, Z^{(2|1)}] &= p M^{(8)} + \tilde{p} Z^{(8)} + q M^4 + \tilde{q} W^{(5)} + r Z^{(4|2)}, \\
[Z^{(2|1)}, Z^{(4|2)}] &= \rho \left( Z^{(2|1)} - \frac{1}{4!} \epsilon_8 Z^{(2|1)} \right) + \sigma \epsilon_5 Z^{(4|2)}, \\
[Z^{(4|2)}, Z^{(4|2)}] &= (\delta - \frac{2}{4!} \epsilon) (\lambda M^{(8)} + \tilde{\lambda} W^{(8)}) + (\delta - \frac{1}{4!} \epsilon) (\kappa M^{(5)} + \tilde{\kappa} W^{(5)}) \\
&\quad + \chi (\delta - \frac{1}{4!} \epsilon) Z^{(2|1)} + \phi Z^{(4|2)}.
\end{aligned} \tag{A.11}$$

The structure involving the  $\delta$  and  $\epsilon$  parts is determined by imposing self-duality on the tensors on the left-hand side; explicit indices can easily be inserted.

The parameters  $a$  and  $b$  are fixed to be  $a = 1$  and  $b = -2$  because we want the structure of the  $\text{osp}^*(8|4)$  algebra to be manifestly present in the  $\{Q, Q\}$  bracket. In order to determine the remaining parameters, we first scale the generators  $W^{(8)}$ ,  $W^{(5)}$  and  $Z^{(1|1)}$  in such a way that

$$\tilde{\beta} = 1, \quad \tilde{\delta} = 1, \quad p = \pm 1, \tag{A.12}$$

which is always possible. The  $(M, M, W)$  Jacobi identities restrict the way in which the two-form charges act on each other; from  $(M^{(8)}, M^{(8)}, W^{(8)})$  one finds

$$\{\alpha = 0, \tilde{\alpha} = 1\} \quad \text{or} \quad \tilde{\alpha} = 0. \tag{A.13}$$

Similar conditions for  $\gamma$  and  $\tilde{\gamma}$  arise from  $(M^{(5)}, M^{(5)}, W^{(5)})$ . We discard the second option because it makes the interpretation of the  $M$  charges as isometry generators troublesome. The action of these isometry generators on the “standard” non-perturbative charges  $Z^{(2|1)}$  and  $Z^{(4|2)}$  is fixed by imposing the following Jacobi identities:

$$\begin{aligned}
(M^{(8)}, M^{(8)}, Z^{(2|1)}) &\rightarrow m = 1, & (M^{(5)}, M^{(5)}, Z^{(2|1)}) &\rightarrow n = \frac{1}{2}, \\
(M^{(8)}, M^{(8)}, Z^{(4|2)}) &\rightarrow \mu = 2, & (M^{(5)}, M^{(5)}, Z^{(4|2)}) &\rightarrow \nu = 1.
\end{aligned} \tag{A.14}$$

One now first fixes the action of all two-form charges on the fermions, using for instance the Jacobi identities

$$\begin{aligned} (M^{(8)}, Q, Q) &\rightarrow A = \frac{1}{8}, & (M^{(5)}, Q, Q) &\rightarrow B = \frac{1}{8}, \\ (W^{(8)}, Q, Q) &\rightarrow A_z = \frac{\tilde{m}}{8} = \frac{\tilde{\mu}}{8}, & (W^{(5)}, Q, Q) &\rightarrow B_z = \frac{\tilde{n}}{8} = \frac{\tilde{\nu}}{8}. \end{aligned} \quad (\text{A.15})$$

The structure of the algebra in the sector of the two-form charges is then fixed to a large extent from the Jacobi identities

$$\begin{aligned} (W^{(8)}, W^{(8)}, Q) &\rightarrow \beta = \tilde{m}(\tilde{m} - 1), \\ (W^{(5)}, W^{(5)}, Q) &\rightarrow \delta = \tilde{n}(\tilde{n} - 1), \end{aligned} \quad (\text{A.16})$$

and using a different part of the  $(Z, Q, Q)$  identities,

$$\begin{aligned} (W^{(8)}, Q, Q) &\rightarrow \tilde{a} = \frac{1}{\tilde{m} - 1}, \\ (W^{(5)}, Q, Q) &\rightarrow \tilde{b} = \frac{-2}{\tilde{n} - 1}. \end{aligned} \quad (\text{A.17})$$

A key relation between the two remaining parameters  $\tilde{m}$  and  $\tilde{n}$  is found from

$$\begin{aligned} (Z^{(2|1)}, Q, Q) &\rightarrow \frac{2\tilde{m} - 1}{\tilde{m} - 1} = 8cC = -2\frac{2\tilde{n} - 1}{\tilde{n} - 1}, \\ (Z^{(4|2)}, Q, Q) &\rightarrow cC = 2 \cdot 4!dD. \end{aligned} \quad (\text{A.18})$$

This last line ensures that the triple- $Q$  identity is satisfied, since it leads to

$$(Q, Q, Q) \rightarrow 2(aA + \tilde{a}A_z) + (bB + \tilde{b}B_z) + 2cC - 10 \cdot 4!dD. \quad (\text{A.19})$$

Finally, one determines the way in which the  $W$  charges appear in the  $[Z^{(2|1)}, Z^{(2|1)}]$  bracket by considering

$$\begin{aligned} (Z^{(2|1)}, Z^{(2|1)}, W^{(8)}) &\rightarrow \tilde{p} = \frac{1}{\tilde{m} - 1}, \\ (Z^{(2|1)}, Z^{(2|1)}, W^{(5)}) &\rightarrow \tilde{q} = \frac{1}{\tilde{n} - 1}, \\ (Q, Q, Z^{(2|1)}) &\rightarrow q = \mp 1, \end{aligned} \quad (\text{A.20})$$

(the sign in the last equation depending on the sign of  $p$  in (A.12)). Analogously one determines similar coefficients in the  $[Z^{(4|2)}, Z^{(4|2)}]$  bracket from

$$(Q, Q, Z^{(4|2)}) \rightarrow (\tilde{m} - 1)\tilde{\lambda} = \lambda, \quad (\tilde{n} - 1)\tilde{\kappa} = \kappa. \quad (\text{A.21})$$

The Jacobi identities not listed so far do not impose any further restrictions on the parameters. At this stage, one can now diagonalise the algebra in the two-form sector, as explained in the main text, and obtain the structure given in (2.13).



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