



International School for Advanced Studies
Mathematical Physics Sector

BOSE-EINSTEIN CONDENSATION: ANALYSIS OF PROBLEMS AND RIGOROUS RESULTS

BY

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