

WARPED MODELS IN STRING THEORY

B.S. ACHARYA^a, F. BENINI^b AND R. VALANDRO^b

^a Abdus Salam ICTP, Strada Costiera 11,
34014 Trieste, ITALIA

and

INFN, sezione di Trieste

bacharya_at_cern.ch

^b International School for Advanced Studies (SISSA/ISAS),
Via Beirut 2-4, 34014 Trieste, ITALIA

benini, valandro_at_sissa.it

ABSTRACT

Warped models, originating with the ideas of Randall and Sundrum, provide a fascinating extension of the standard model with interesting consequences for the LHC. We investigate in detail how string theory realises such models, with emphasis on fermion localisation and the computation of Yukawa couplings. We find, in contrast to the 5d models, that fermions can be localised *anywhere* in the extra dimension, and that there are *new mechanisms* to generate exponential hierarchies amongst the Yukawa couplings. We also suggest a way to distinguish these string theory models with data from the LHC.

1 Introduction

Since the seminal papers of Randall and Sundrum [1], models of particle physics constructed in a five dimensional warped geometry have offered insights into the hierarchy problem, fermion masses and many other issues related to the standard model. Since the original ideas proposed in [1], the state of the art 5d models have evolved somewhat [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18] (see, for instance, [19] for a review). Moreover, there are potentially very interesting signals for the LHC, since these models are dual descriptions of ‘compositeness’ [20]. Their most basic features are:

- a) for every standard model field, there is a bulk 5d field;
- b) to solve the hierarchy problem, the Higgs is localised in a region of large warping;
- c) turning on bulk and boundary masses localises the fermion zero modes and hence one obtains hierarchical Yukawa couplings since the fermions can have varying degrees of overlap with the Higgs.

Since these models have arbitrary parameters *e.g.* the bulk and boundary masses, we would like to investigate the realisation of these models in string theory. This perspective offers a framework for explaining the parameters of the 5d models and some new insights:

- To realise a warped geometry we consider warped string compactifications which arise naturally in the IIB string theory with fluxes [21, 22].
- Matter and gauge fields in the bulk arise as strings which end on $D7$ -branes in the bulk¹.
- To have several standard model generations, we turn on a topologically non-trivial (“instanton”) background field on the $D7$ worldvolume.
- Fermion zero modes then naturally localise near the instantons and/or by warping. This leads to new features: a) the zero modes can be *localised anywhere* in the extra dimension and b) the scale of the topologically non-trivial background (instanton size) can also be used to suppress Yukawa couplings, in addition to the usual mechanism of separating the fermion zero modes in the extra dimension.

¹ Previous studies of warped models in string theory have tended to have the standard model on $D3$ -branes [23] See however [24].

Our main results will be explicit formulae for the profile of the fermion zero modes in the 5th dimension and their Yukawa couplings. These formulae show in particular how the physical size of the topologically non-trivial “instanton” background field can give rise to hierarchies of Yukawa couplings. They also show that the large Yukawa coupling is associated with a “small instanton” in the extra dimension. In the next section, which is relatively self-contained, we will illustrate these results in the simplest possible example. Following this we go on to describe more general results including examples in which there can be several “instantons” in the background, leading to new effects. We conclude the paper with a discussion of the differences between the string theory vacua and the 5d models and suggest a way in which the LHC data could be used to distinguish amongst them. The appendices deal with technicalities on the fermion zero modes and the Dirac operator.

2 A Simple Example

In this section we will describe a simple example which illustrates most of the features of the more general calculations that are given in the rest of the paper.

Our interest is understanding how various features of the 5d phenomenological models are realised in string theory vacua, with the motivation that this might lead to additional insights about the phenomenology. The three basic features which we wish to understand better are:

- a) The 5d warped models tend to have the standard model gauge fields propagating in the bulk of AdS_5 .
- b) For each standard model fermion there is a 5d bulk fermion field with both bulk and boundary mass parameters which determine whether or not the fermion is localised in the UV or IR end of AdS_5 .
- c) The hierarchy amongst standard model Yukawa couplings is realised by the varying degrees of overlap between these localised wavefunctions and the Higgs.

We will study the string theory realisation of these features within the context of Type IIB string theory vacua with fluxes, since this class of vacua realises warped extra dimensions in a natural way. In such vacua, non-Abelian gauge fields can reside on $D3$ and $D7$ -branes, so in order to realise property a) the only possibility is to put the standard model gauge fields on the $D7$ -branes. Recall that the ten dimensional spacetime is a warped product of four-dimensional Minkowski spacetime $M^{3,1}$ and a compact Calabi-Yau manifold Z [21]. The metric takes the form of a $D3$ -brane metric, where the $D3$ -branes span the Minkowski spacetime. The $D7$ -branes have a world-volume which is a warped product of $M^{3,1}$ and a four dimensional cycle $\Sigma \subset Z$.

Now we turn to property b). The physics behind the introduction of bulk and boundary masses is that, before symmetry breaking, the standard model fermions are all zero modes of the Dirac operator on $M^{3,1}$. We thus need to study the Dirac equation on the $D7$ -brane in the warped background. For the ten dimensional geometries described in [21] the metric induced on the $D7$ -branes is of the form:

$$ds_8^2 = f(z)^{-1/2} \eta_{\mu\nu} dx^\mu dx^\nu + f(z)^{1/2} g_{\alpha\beta} dz^\alpha dz^\beta \quad (\alpha, \beta = 1, \dots, 4), \quad (2.1)$$

where the warp factor f is a function of the coordinates z^α on the 4-cycle Σ , which the $D7$ -brane wraps and x^μ are coordinates on $M^{3,1}$. For simplicity,

we will study the warped geometry induced by $D3$ -branes in flat spacetime. Most of the interesting features we observe are not very sensitive to the geometry of Σ as we will see. In this case

$$ds_8^2 = f(r)^{-1/2} \eta_{\mu\nu} dx^\mu dx^\nu + f(r)^{1/2} \delta_{\alpha\beta} dz^\alpha dz^\beta \quad (\alpha, \beta = 1, \dots, 4), \quad (2.2)$$

where $f(r) = 1 + L^4/(r^2 + d_0^2)^2$, $r^2 = |\vec{z}|^2$ and d_0 is the separation between the $D7$ and the $D3$ -branes. For simplicity, in this example, we will set $d_0 = 0$.

We will also use an almost “flat” radial coordinate y defined by

$$r = L e^{-ky} \quad k = \frac{1}{L} \quad (2.3)$$

For illustration, the near horizon geometry in these coordinates is

$$ds_8^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 + L^2 d\Omega_3^2 \quad (2.4)$$

$$(2.5)$$

which is an $AdS_5 \times S^3$ contained in $AdS_5 \times S^5$. In these coordinates, $y \rightarrow \infty$ is the tip of the throat while $y = 0$ is its origin.

The low energy spectrum of the $D7$ -brane modes includes massless fermions in the adjoint representation of the gauge group: $\mathcal{D}_8 \Psi = 0$. Under the splitting induced by the $D3$ -brane background, the fermions factorise as products of fermions on $M^{3,1}$ and $\Sigma = \mathbb{R}^4$:

$$\Psi = \sum_k \chi_k(x) \otimes d_{\psi_k} \psi_k(z), \quad (2.6)$$

where d_{ψ_k} is a normalization constant.

The Dirac equation can be written as²

$$\mathcal{D}_8 \Psi = \left(f^{1/4} \tilde{\mathcal{D}}_{3,1} + \frac{1}{f^{1/4}} \tilde{\mathcal{D}}_4 - \frac{1}{8f^{1/4}} \frac{f'}{f} \gamma_r \right) \Psi = 0, \quad (2.7)$$

where $\tilde{\mathcal{D}}_{3,1}$ and $\tilde{\mathcal{D}}_4$ are respectively the Dirac operator on $M^{3,1}$ and on flat \mathbb{R}^4 . Massless fermions in $M^{3,1}$ are the zero modes of $(\tilde{\mathcal{D}}_4 - \frac{f'}{8f} \gamma_r)$. As shown in the appendix these are given by:

$$\psi = f^{1/8} \tilde{\psi},$$

where $\tilde{\psi}$ are the zero modes of the operator $\tilde{\mathcal{D}}_4$. This means that in the warped background, the *four-dimensional zero modes are conformally equal to the zero modes in an unwarped geometry*.

²In our conventions, Γ^μ are the gamma matrices relative to the background metric, while γ^μ are relative to the flat metric.

The simplest possibility in this example is to take $\tilde{\psi}$ to be the constant zero modes of the flat Euclidean Dirac operator $\tilde{\mathcal{D}}_4$ on the extra dimensions. Whilst this indeed will give us a four dimensional fermion zero mode, it raises two problems:

1. since the fermion field Ψ on the $D7$ -brane is in the adjoint representation, the four dimensional zero mode ψ is also in the adjoint representation;
2. there will be *four* such fermion zero modes (since there are four constant spinors), whilst the standard model requires three generations of zero modes in representations which are certainly not adjoint.

In principle, there is an elegant solution to both of these problems, which also elucidates the string theory description of property *b*): the gauge covariant Dirac operator $\tilde{\mathcal{D}}_4$ can have multiple non-trivial zero modes in the presence of topologically non-trivial gauge field backgrounds. This is a standard mechanism to generate light fermion generations in string theory, however the novelty here is the presence of the warp factor in ψ and that we will be quite explicit about the profile of the wavefunction.

As is well known from gauge theory instanton physics, gauge field-strengths satisfying the condition $F = - * _4 F$ in four Euclidean dimensions (usually called instantons) can be topologically non-trivial and support multiple fermion zero modes which are not in the adjoint representation. Depending on the topological charge (or instanton number) one can have different numbers of fermion zero modes. One can check that such gauge field configurations also solve the equations of motion on the $D7$ -brane, so are acceptable backgrounds.

The zero mode wave functions $\tilde{\psi}$ have been computed explicitly long ago for many different $F = - * _4 F$ backgrounds [25]. If we take the simplest known solution to these equations [32], then we obtain a zero mode which depends on the *size* of the instanton ρ , as well as its position \vec{Z}_ψ in the Euclidean space:

$$\psi(\vec{z}) = f^{1/8} \frac{\rho}{[\rho^2 + (\vec{z} - \vec{Z}_\psi)^2]^{3/2}} \eta, \quad (2.8)$$

here η is a constant spinor normalized as $\eta^\dagger \eta = 1$.

These fermion zero modes have to be normalised properly. Consider the

kinetic term:

$$\begin{aligned}
& - \int d^8x \sqrt{-G} G^{\mu\nu} \bar{\Psi} \Gamma_\mu \partial_\nu \Psi + \dots \\
& = - \int d^4x \eta^{\mu\nu} \bar{\chi}(x) \gamma_\mu \partial_\nu \chi(x) \int d^4z d_\psi^2 f^{1/4}(z) \psi(z)^\dagger \psi(z) + \dots \quad (2.9)
\end{aligned}$$

where the normalisation constant d_ψ was introduced in the Kaluza-Klein ansatz (2.6) and we used $\Gamma_\mu = f^{-1/4} \gamma_\mu$. In order to have a canonical kinetic term, we require:

$$d_\psi^2 \int d^4z f^{1/4} \psi^\dagger \psi = 1 \quad (2.10)$$

In regions of negligible warping, this condition is realised for $d_\psi \sim 1$, whilst when the warp factor is large (for instance in the near horizon region) the normalisation is given by:

$$d_\psi^{-2} = \int r^3 dr 4\pi \sin \theta d\theta f(r)^{1/2} \frac{\rho^2}{(\rho^2 + r^2 + Z_\psi^2 - 2rZ_\psi \cos \theta)^3} \quad (2.11)$$

$$= \frac{\pi^2}{2} \left(\frac{\rho^2}{L^2} + e^{-2kY_\psi} \right)^{-1}, \quad (2.12)$$

where $|\vec{Z}_\psi|/L \equiv e^{-kY_\psi}$ is the radial position of the instanton in almost flat radial coordinates. When $\rho/L < e^{-kY_\psi}$, we get $d_\psi \simeq (\sqrt{2}/\pi) e^{-kY_\psi}$.

We see that in string theory the instanton scale size is important in determining the profile of the fermion zero modes. Putting all the factors together, the *normalised* zero mode wave function is:

$$d_\psi \psi \sim e^{-kY_\psi} e^{\frac{k}{2}y} \frac{\rho}{\left[\rho^2 + (\vec{z} - \vec{Z}_\psi)^2 \right]^{3/2}} \eta \quad (2.13)$$

We can compare this wave function with the 5d models:

$$d_{\psi 5d} \psi_{5d} \sim \sqrt{\frac{k(1-2c)}{e^{(1-2c)kR} - 1}} e^{(2-c)ky}, \quad (2.14)$$

where the constant c is a combination of bulk and boundary masses.

From this we learn that the zero mode wavefunction in string theory is quite different from the 5d models. Note that there is a dependence on the instanton scale size, ρ . In particular, in string theory *the zero mode can be localised anywhere* in the 5th dimension.

2.1 Instantons as $D3$ -branes

As is well known, gauge field backgrounds on $D7$ -branes with $F \wedge F \neq 0$ carry $D3$ -brane charge [26]. In fact, smooth instanton backgrounds such as those we are considering here, are “fat $D3$ -branes” with size ρ . Therefore, we can also say that the fermion zero modes are localised on fat $D3$ -branes. The fermion zero modes are therefore 3-7 strings. Note however that, in order to trust the metric we have been using, we should consider the number of such fat $D3$ -branes to be small compared to the large number of ordinary $D3$ -branes and fluxes which generate the bulk geometry.

The parameters ρ and \vec{Z}_ψ are therefore moduli field vevs which arise in the open string sector. It would be interesting to investigate mechanisms which stabilise these moduli. Presumably closed and open string fluxes generate a potential for these fields.

2.2 Yukawa Couplings

The zero mode profiles are crucial for computing the 4d Yukawa couplings, and clearly the answer will depend on ρ . In order to determine the Yukawa couplings, we need to identify the Higgs field in string theory. Essentially, with only $D3$ and $D7$ -branes the Higgs can be a 7-7 or a 3-7 string, since it must be charged under the standard model gauge group. The simplest case to consider is that the Higgs is a 3-7 string state. The 7-7 case will be described later. In this case its wavefunction will be localised near a point \vec{Z}_H in Σ and we will simply model this by a delta-function. This choice is very similar to the standard 5d proposal [19].

We must first determine the correctly normalised 4d Higgs field from its kinetic term by imposing

$$\begin{aligned} - \int d^8x \sqrt{-\hat{G}_{3,1}} G^{\mu\nu} d_H^2 \partial_\mu \bar{H}(x) \partial_\nu H(x) \delta(\vec{z} - \vec{Z}_H) &= \\ &= - \int d^4x \partial_\mu \bar{H}(x) \partial^\mu H(x) \quad (2.15) \end{aligned}$$

which gives $d_H = f(|\vec{Z}_H|)^{1/4}$.

The four dimensional Yukawa coupling is obtained by direct dimensional

reduction of the 8d one (remembering localisation of the Higgs):

$$\begin{aligned} \int d^8x \sqrt{-\hat{G}_{3,1}} \lambda^{(8)} d_H \bar{\Psi} \Psi H \delta(\vec{z} - \vec{Z}_H) = \\ = \lambda^{(8)} d_H d_\psi^2 f(|\vec{Z}_H|)^{-1} \psi(\vec{Z}_H)^2 \int d^4x \bar{\chi}(x) \chi(x) H(x), \end{aligned} \quad (2.16)$$

so that

$$\lambda = \lambda^{(8)} d_\psi^2 \left. \frac{\psi^2(z)}{f(z)^{3/4}} \right|_{\vec{Z}_H}. \quad (2.17)$$

Remember that the 8d Yukawa has dimension of (length)⁴. We see therefore that the Yukawa coupling in the standard model is determined by several factors: the fermion zero mode evaluated at the Higgs position, the warp factor at the Higgs position and the normalisation constant d_ψ (which itself depends on ρ and Y_ψ).

Let us analyse the 4d Yukawa coupling further. For simplicity we will study the case when the fermion zero mode is localised in a region of large warping and $\rho/L < e^{-kY_\psi}$. Then the 4d Yukawa coupling is given by:

$$\lambda = \frac{2}{\pi^2} \lambda^{(8)} e^{-2k(Y_H + Y_\psi)} \frac{\rho^2}{[\rho^2 + (\vec{Z}_H - \vec{Z}_\psi)^2]^3},$$

where again we used almost flat radial coordinates $|\vec{Z}_H|/L \equiv e^{-kY_H}$. In the standard model the Yukawa couplings of the charged fermions range from order one for the top quark to 10^{-6} for the electron, and clearly (2.18) is rich enough to span this range. In more detail, the top quark Yukawa coupling ($\lambda \sim 1$) can arise when the top wave function peaks at the location of the Higgs *i.e.* $Y_H = Y_\psi$:

$$\lambda = \frac{2}{\pi^2} \frac{\lambda^{(8)}}{\rho^4} e^{-4kY_H} \quad (2.18)$$

Notice that, due to the warping in the spacetime, ρ is not the physical size ρ_{phys} of the instanton, which depends upon its location in AdS_5 :

$$\rho_{\text{phys}} = \int_{|\vec{Z}_\psi| - \frac{\rho}{2}}^{|\vec{Z}_\psi| + \frac{\rho}{2}} ds = \int_{|\vec{Z}_\psi| - \frac{\rho}{2}}^{|\vec{Z}_\psi| + \frac{\rho}{2}} f^{1/4}(r) dr \simeq e^{kY_\psi} \rho, \quad (2.19)$$

where the last result is valid when $\rho < Le^{-kY_\psi}$. The same can be seen by evaluating the instanton displacement in the almost flat radial coordinate: $\Delta y = e^{kY_\psi} \rho$. Note that in terms of the physical size, this is simply $\rho_{\text{phys}} < L$:

the instanton is physically smaller than the AdS_5 radius, which is a natural requirement. Substituting in (2.18), one gets:

$$\lambda = \frac{2}{\pi^2} \frac{\lambda^{(8)}}{\rho_{\text{phys}}^4}. \quad (2.20)$$

In general, we expect $\lambda^{(8)}$ to be of order ℓ^4 , with ℓ the string scale, we obtain $\lambda \sim 1$ when ρ_{phys} is of order of the string scale. In other words, *the instanton which localises the top quark is a small instanton*. We therefore might expect strong quantum corrections to the top sector. On the other hand, when ρ_{phys} is larger than the fundamental scale, λ is smaller than 1 and we can also realize smaller Yukawa couplings by localising the corresponding fermions on large instantons.

The smaller Yukawa couplings are actually better obtained in the case when $Y_\psi < Y_H$, which means that the fermion zero mode is localised far from the Higgs, and again when $\rho < Le^{-kY_\psi}$. The Yukawa coupling is then given by:

$$\lambda = \frac{2}{\pi^2} \lambda^{(8)} e^{-2k(Y_H - 2Y_\psi)} \frac{\rho^2}{L^6}. \quad (2.21)$$

This can be written as

$$\lambda = \frac{2}{\pi^2} \frac{\lambda^{(8)}}{\rho_{\text{phys}}^4} \frac{\rho_{\text{phys}}^6}{L^6} e^{-2k(Y_H - Y_\psi)}. \quad (2.22)$$

We see that even when the AdS_5 radius L is just a little bigger than the instanton size, that *both the instanton scale size and the warp factor suppress the generic Yukawa coupling*.

3 The Higgs as a Vector Zero Mode

In this section we will study the case that the Higgs is a 7-7 string which is a zero mode of the 8-dimensional gauge field on the $D7$ -brane. We will see that such zero modes are not affected by the presence of the warping and can also be computed in the instanton background.

In the 8-dimensional kinetic term, all the fields are in the adjoint representation of the gauge group G . The background instanton gauge field breaks this group, leaving a (3+1)-dimensional gauge theory, whose gauge group is a subgroup G' of G . The adjoint representation of G splits into irreps of $G' \times SU(2)$, where $SU(2)$ is chosen as the gauge group of the instanton. Thus, an 8-dimensional field in the Adj rep of G can be written as a sum of

products of fields in $M^{3,1}$ and Σ in various representations of $G' \times SU(2)$. In order to reproduce a GUT theory at low energy, we could take G' to contain some GUT group as a subgroup.

Let us see some details. The 8-dimensional kinetic term is:

$$\int d^8 X \sqrt{-G} \bar{\Psi} \not{D} \Psi \quad (3.1)$$

and contains the term

$$g \int d^8 X \sqrt{-G} \bar{\Psi} \not{A} \Psi \supset g \int d^4 x \bar{\chi}_i(x) \chi_j(x) H_k(x) \int d^4 y \psi_i^\dagger(y) \not{A}_k(y) \psi_j(y)$$

where g is the 8d gauge coupling (of order ℓ^2 , with ℓ the string length) and where we have used the splitting (2.6) of the fermion fields and that of the vector:

$$A(x, y)_m dy^m = A_{bk}(y) + \sum_k H^k(x) \delta a_k(y) . \quad (3.2)$$

We see that the effective Yukawa coupling in $(3+1)$ -dimensions is given by:

$$g d_{\psi_i} d_{\psi_j} d_H \int d^4 y \tilde{\psi}_i^\dagger(y) \not{A}_k(y) \tilde{\psi}_j(y) . \quad (3.3)$$

where we have substituted the expression (2.8) for the fermion zero modes ψ . Note that the warp factor has disappeared; it only enters in the fermion normalisation constants³. The zero modes $\delta a_k(y)$ are warp factor independent because the Yang-Mills action on Σ is conformally invariant.

The fields ψ_i , ψ_j and δa_k are in the $SU(2)$ representations dictated by the splitting of $\text{Adj}G$ and by the G' representations that one wants H , χ_i and χ_j to belong to.

We will compute the integral (3.3) in the simple case in which the two fermions are in the fundamental representation of $SU(2)$, while the vector zero mode is in the adjoint. We will see that the coupling can be highly suppressed in the usual approximation of well separated instantons, and that this suppression is due to the localisation of the zero modes near individual single instantons. This justifies this simple choice of representations, since the localisation is characteristic of the zero modes in any representation. This is important, because the suppression works whatever $SU(2)$ -representations are associated (by the splitting of $\text{Adj}G$) with the particular GUT-representations that one wants to find in the GUT Yukawa interaction

³For this particular choice for the Higgs, its normalisation is not affected by the warping and will be put $d_H = 1$

terms. It would be interesting to compute the integral exactly, since new phenomena might arise.

We will consider the 't Hooft solution with instanton number $k = 2$. This solution has $5k = 10$ explicit parameters: ρ_1 , ρ_H , \vec{Z}_1 and \vec{Z}_H . The zero mode profiles when $k > 1$ are given in the appendix. We will also choose both the fermion zero modes in (3.3) to be localised around \vec{Z}_1 , while the vector one (the Higgs) is to be localised around \vec{Z}_H . We put \vec{Z}_H in a region of large warping, in order to address the hierarchy problem. We will see that, in order to have a sufficiently large top Yukawa coupling, one must have δa sharply localised around \vec{Z}_H .

We substitute the expressions (A.22) and (A.25) in (3.3) and estimate it in several asymptotic regions of the parameter space of the $k = 2$ solution. With more than one instanton, we will find a new suppression mechanism: due to the localisation of wavefunctions at well separated points, suppression can also occur due to a hierarchy in the two instanton sizes ρ_1 and ρ_H . The maximal value of the integral is actually obtained when $|\vec{Z}_H - \vec{Z}_1| \ll \rho_1, \rho_H$ and $\rho_1 \sim \rho_H$.

Actually when $|\vec{Z}_H - \vec{Z}_1| \ll \rho_1, \rho_H$, the parameter $X \equiv |\vec{Z}_1 - \vec{Z}_H|$ disappears from the result, that is:

$$g d_\psi^2 \int \psi_i^\dagger \sigma^\mu \delta A_\mu^\Phi \psi_j \simeq g d_\psi^2 \alpha^\Phi \int r^3 dr \frac{\rho_1^2 \rho_H^2}{(r^2 + \rho_1^2 + \rho_H^2)^4} = d_\psi^2 \frac{g \alpha^\Phi}{24} \frac{\rho_1^2 \rho_H^2}{(\rho_1^2 + \rho_H^2)^3} \quad (3.4)$$

where δA_μ^Φ is defined in (A.25), and where α^Φ is a constant of order one. The expression (3.4) takes its maximal value when $\rho_1 \sim \rho_H$:

$$g d_\psi^2 \int \psi_i^\dagger \sigma^\mu \delta a_\mu^\Phi \psi_j \sim \frac{2}{\pi^2} \frac{g}{\rho_H^2} e^{-2\kappa Y_H} \quad (3.5)$$

The same result as (3.5) is obtained taking $k = 1$. Then one has to substitute the physical size in this formula (see (2.19)). The final result is:

$$\lambda = \frac{2}{\pi^2} \frac{g}{\rho_{H\text{phys}}^2} \quad (3.6)$$

From here, we see that if one wants the top coupling to be of order one, the top zero mode must be localised close to the Higgs and the ρ -parameter of the corresponding instanton has to be of the order of the Higgs one.

The Yukawa hierarchy can then be obtained by varying the instanton parameters in such a way as to have different overlaps of the zero modes.

One can approximate the integral giving the Yukawa couplings in different asymptotic regions of the instanton moduli space. We summarize the results in Table 1. In order to get the actual Yukawa coupling, this integral has to be multiplied by d_ψ^2 and the instanton ‘sizes’ have to be substituted with their physical sizes. Let us consider some relevant cases, which turn out to be similar to the result found in the simple example of the previous section.

- When the fermions are localised around the same position of the Higgs:

$$\lambda = \frac{g}{\rho_{\psi\text{phys}}^2} \left(\frac{\rho_H}{\rho_\psi} \right)^2$$

- When the fermions are far from the Higgs:

$$\begin{aligned} \frac{X}{\rho_\psi} \frac{\rho_H^2}{\rho_\psi^2} \gg 1 &\rightarrow \lambda = \frac{g}{\rho_{\psi\text{phys}}^2} \left(\frac{\rho_\psi}{X} \right)^4 \\ \frac{X}{\rho_\psi} \frac{\rho_H^2}{\rho_\psi^2} \ll 1 &\rightarrow \lambda = \frac{g}{\rho_{\psi\text{phys}}^2} \left(\frac{\rho_\psi}{X} \right)^3 e^{-2\kappa(Y_\psi - Y_H)} \end{aligned}$$

limits	$g \int d^4z \tilde{\psi}_i^\dagger(z) \Phi_H(z) \tilde{\psi}_j(z)$
$\rho_H \sim \rho_\psi \ll X$	$\frac{g}{\rho_H^2} \left(\frac{\rho_H}{X} \right)^3$
$\rho_H \ll \rho_\psi \sim X$	$\frac{g}{\rho_H^2} \left(\frac{\rho_H}{X} \right)^2$
$\rho_H \ll \rho_\psi \ll X$	$\frac{g}{\rho_H^2} \left(\frac{\rho_H}{X} \right)^2 \left(\frac{\rho_\psi}{X} \right)^2 \left[1 + \frac{X}{\rho_\psi} \left(\frac{\rho_H}{\rho_\psi} \right)^2 \right]$
$\rho_H \ll X \ll \rho_\psi$	$\frac{g}{\rho_H^2} \left(\frac{\rho_H}{\rho_\psi} \right)^4 \left[1 + \left(\frac{X}{\rho_H} \right)^2 \left(\frac{X}{\rho_\psi} \right)^2 \right]$
$X \lesssim \rho_H \ll \rho_\psi$	$\frac{g}{\rho_H^2} \left(\frac{\rho_H}{\rho_\psi} \right)^4$

Table 1: Various limits of the integral giving the Yukawa coupling.

4 Comments

We have seen that there is a rather intricate string theory picture underlying many of the important features of the 5d warped phenomenology models. One might ask: can we distinguish the string theory models from the 5d phenomenology?

Obviously, yes in principle: the spectrum of the 5d models consists of the zero modes which become the standard model particles after symmetry breaking; then in addition, for each standard model particle there is an infinite Kaluza-Klein tower of resonances with the *same spin* as its associated standard model cousin. These particles are also present in the string spectrum, but the string theory has more: for each standard model particle, there is also an *infinite tower of string states of increasing spins*. So, measuring even part of the spectrum could be enough to distinguish them.⁴

In the 5d models, the masses of the Kaluza-Klein modes are typically quantised in units of a TeV. Therefore, the LHC will only be sensitive to the first or second resonance. What about the string states? The AdS_5 scale is of order m_p so, for weak string coupling the string scale is below this. However, the $D7$ -branes fill the entire AdS_5 and hence, the $7-7$ strings which are in the infrared end of AdS_5 will have a TeV scale or lower mass: hence only the first or second of these will be directly accessible at the LHC. Since these states have the same gauge quantum numbers as the Kaluza-Klein modes, they could only be distinguished by their decay patterns or their spins. For example, there might be a spin $3/2$ colored particle which is a string excitation of the gluon. If produced, this particle must eventually decay into jets, the angular distributions of which will be sensitive to its spin. It is interesting to study to what extent these events can be selected and the discovery reach for the LHC [28].

We conclude with a discussion of some additional issues which deserve further investigation. Firstly there is the issue of supersymmetry breaking. In 5d models, one does not a priori need supersymmetry at all, since the electroweak scale is generated through the warped extra dimension. But in Type IIB string theory, there is certainly local supersymmetry in the UV, and one needs to break it. One possibility is to choose the background fluxes and geometry to explicitly break supersymmetry, such as was recently considered in [29]. However, backgrounds which explicitly break supersymmetry in string theory can often be unstable; thus, it would be good to investi-

⁴Usually, in the holographic limit [27] we decouple these massive open string states, but here we cannot since the string length and string coupling is finite.

gate this further. Secondly, there is the issue of fermion chirality. With one collection of parallel $D7$ -branes, even though the backgrounds we have considered generate multiple copies of the same standard model representations, the representations include both fermion chiralities. This can be avoided by the introduction of another set of $D7$ -branes intersecting the first set along a surface in Σ , but we have not investigated this in detail. Also, in 5d models, the chirality problem is resolved by considering a \mathbb{Z}_2 orbifold and perhaps such a mechanism can also be realised in string theory.

Acknowledgments

We would like to thank Joseph Conlon, Paolo Creminelli, Michael R. Douglas, Johanna Erdmenger, Shamit Kachru, Maurizio Piai and Marco Serone for useful and stimulating discussions.

A The ADHM Construction

In this section we briefly review the ADHM formalism for instantons and how to use it to find bosonic and fermionic zero modes around their background [30] (to have a more complete review of the subject, see [31] and references therein). We are interested in constructing finite action solutions of the four dimensional Euclidean Yang-Mills theory (instantons). The gauge potential satisfies a first order (anti-)self-duality equation

$$F_{\mu\nu} = \pm(*F)_{\mu\nu} = \pm\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F_{\rho\sigma} \quad (\text{A.1})$$

In the following we will restrict ourselves to $U(N)$ gauge groups.

In order to discuss the ADHM formalism, we introduce the quaternionic notation:

$$\mathbf{z} = z_\mu\sigma_\mu \quad \bar{\mathbf{z}} = z_\mu\bar{\sigma}_\mu \quad (\text{A.2})$$

$$z_\mu = \frac{1}{2}\text{tr } \mathbf{z} \bar{\sigma}_\mu \quad (\text{A.3})$$

where $\sigma_\mu = (i\tau^a, 1)$ and $\bar{\sigma}_\mu = (-i\tau^a, 1)$.

The ADHM formalism allows to obtain (anti-)self-dual field strength configurations by solving only algebraic equations. The gauge field with instanton number k for $U(N)$ gauge group is given by

$$A_\mu = v(z)^\dagger \partial_\mu v(z), \quad (\text{A.4})$$

where $v(z)$ is a $(N + 2k) \times N$ matrix. It is defined by the equations

$$v(z)^\dagger v(z) = 1 \quad (\text{A.5})$$

$$v(z)^\dagger \Delta(z) = 0. \quad (\text{A.6})$$

Here $\Delta(z)$ is a $(N + 2k) \times 2k$ matrix, linear in the position variable z , having the structure

$$\Delta(z) = \begin{cases} a - b\mathbf{z} & \text{self-dual instantons,} \\ a - b\bar{\mathbf{z}} & \text{anti-self-dual instantons,} \end{cases} \quad (\text{A.7})$$

The matrices a, b are constrained to satisfy the condition

$$\Delta(z)^\dagger \Delta(z) = p^{-1}(z) \otimes \mathbb{1}_2 \quad (\text{A.8})$$

where $p^{-1}(z)$ is a $k \times k$ invertible matrix. This assures the (anti-)self-duality equation (A.1).

a, b are $(N + 2k) \times 2k$ matrices that contain the moduli of the instantonic configuration. Because of some symmetries of the equations above they can be brought to the form

$$a = \begin{pmatrix} \lambda \\ \xi \end{pmatrix} \quad b = \begin{pmatrix} 0 \\ \mathbb{1}_{2k} \end{pmatrix}, \quad (\text{A.9})$$

where λ is an $N \times 2k$ matrix and ξ is a $2k \times 2k$. There is no one-to-one correspondence between these two matrices and the moduli: some constraints and redundancies are left. The actual number of moduli is $4Nk$.

Fermion Zero Modes

We will be interested in the fermionic zero modes in the fundamental representation and with definite chirality, *i.e.* those solving:

$$\sigma^\mu D_\mu \eta = \sigma^\mu (\partial_\mu + v^\dagger \partial_\mu v) \eta. \quad (\text{A.10})$$

One gets k independent solutions for η^T as an $N \times 2$ matrix:

$$\eta_{u,\alpha}^i = (v^\dagger b p \sigma^2)_{u,i\alpha} \quad (\text{A.11})$$

where $u = 1, \dots, N$, $i = 1, \dots, k$ and $\alpha = 1, 2$. Thus we have found k fermionic zero modes in the fundamental representation.

A.1 $k = 1$ $SU(2)$ Instanton

We apply the machinery described above to the simplest case of one $SU(2)$ instanton. In this case, applying the further constraints on a and b , one can put Δ in the form:

$$\Delta = \begin{pmatrix} \rho \mathbf{1}_2 \\ \mathbf{Z} - \bar{\mathbf{Z}} \end{pmatrix} \quad (\text{A.12})$$

with ρ and Z_μ the $(4kN - N^2 + 1) = 5$ parameters of the solution in the case $k = 1, N = 2$.

From here, using (A.8), we can get f :

$$p(z) = \frac{1}{\rho^2 + (z - Z)^2} \quad (\text{A.13})$$

Then solving for the normalized zero eigenvectors $v^\dagger \Delta = 0$ and $v^\dagger v = 1$, we have:

$$v(z) = \begin{pmatrix} \left(\frac{(z-Z)^2}{\rho^2 + (z-Z)^2} \right)^{1/2} \mathbf{1}_2 \\ \left(\frac{\rho^2}{(z-Z)^2 (\rho^2 + (z-Z)^2)} \right)^{1/2} (\mathbf{z} - \mathbf{Z}) \end{pmatrix} \quad (\text{A.14})$$

And finally one gets the connection in singular gauge:

$$A_\mu = \frac{\rho^2(z-Z)_\nu}{(z-Z)^2(\rho^2+(z-Z)^2)}\sigma_{\mu\nu} \quad (\text{A.15})$$

Fermion Zero Modes

We compute the fermion zero modes in this simple anti-instanton background, by using the formula (A.11):

$$v^\dagger b p = \frac{\rho}{(\rho^2+(z-Z)^2)^{3/2}} \frac{\mathbf{z}-\mathbf{Z}}{|z-Z|} \quad (\text{A.16})$$

This is a 2×2 matrix. One index is for the fundamental rep, while the other is a spinorial index.

A.2 't Hooft Solution

Now we consider the case in which k is general, the gauge group is $SU(2)$, and we will concentrate on a class of solutions described by $5k$ parameters (instead of $8k$): ρ_i and Z_i , with $i = 1, \dots, k$. It is called the *'t Hooft solution* [32] and is characterized, in the ADHM construction, by:

$$v(z) = \begin{pmatrix} \left[1 + \sum_{i=1}^k \frac{\rho_i^2}{(z-Z_i)^2}\right]^{-1/2} \mathbf{1}_2 \\ \left[1 + \sum_{i=1}^k \frac{\rho_i^2}{(z-Z_i)^2}\right]^{-1/2} \frac{\rho_i^2(\mathbf{z}-\mathbf{Z}_i)}{(z-Z_i)^2} \end{pmatrix} \quad (\text{A.17})$$

It is obtained by taking

$$a = \begin{pmatrix} \rho_i \mathbf{1}_2 \\ \delta_{ji} \mathbf{Z}_i \end{pmatrix} \quad b = \begin{pmatrix} 0 \\ \mathbb{1}_{2k} \end{pmatrix}, \quad (\text{A.18})$$

From these, one can also get the expression for p . The diagonal entries are:

$$p_{ii} = \left[1 + \sum_{\ell=1}^k \frac{\rho_\ell^2}{(z-Z_\ell)^2}\right]^{-1} \frac{1}{(z-Z_i)^2} \left[1 + \sum_{j \neq i} \frac{\rho_j^2}{(z-Z_j)^2}\right], \quad (\text{A.19})$$

while the off-diagonal elements are:

$$p_{ij} = - \left[1 + \sum_{\ell=1}^k \frac{\rho_\ell^2}{(z-Z_\ell)^2}\right]^{-1} \frac{\rho_i \rho_j}{(z-Z_i)^2 (z-Z_j)^2}. \quad (\text{A.20})$$

There are asymptotic regions of the parameters space where the multi-instanton configurations can be identified as being composed of well-separated single instantons. One can show that this limit is valid when

$$(Z_i - Z_j)^2 \gg \rho_i \rho_j \quad \forall i \neq j \quad (\text{A.21})$$

In this limit the Z_i 's become the positions of the k instantons, while the ρ_i 's are their sizes.

Fermion Zero Modes

As in the case $k = 1$, we compute the fermion zero modes in the background described above, by using the formula (A.11):

$$(v^\dagger b p)_h = \left[1 + \sum_{\ell=1}^k \frac{\rho_\ell^2}{(z - Z_\ell)^2} \right]^{-3/2} \frac{\rho_h}{(z - Z_h)^2} \times \left\{ \left[1 + \sum_{\ell=1}^k \frac{\rho_\ell^2}{(z - Z_\ell)^2} \right] \frac{\mathbf{z} - \mathbf{Z}_h}{(z - Z_h)^2} - \sum_{j=1}^k \frac{\rho_j^2}{(z - Z_j)^4} (\mathbf{z} - \mathbf{Z}_j) \right\} \quad (\text{A.22})$$

It is in the fundamental representation of $SU(2)$.

In the limit of well separated k instantons, *i.e.* (A.21), the expression for the fermionic zero modes simplifies:

$$(v^\dagger b p)_h \sim \frac{\rho_h}{(\rho_h^2 + (z - Z_h)^2)^{3/2}} \frac{\mathbf{z} - \mathbf{Z}_h}{|z - Z_h|}. \quad (\text{A.23})$$

It is the same expression for the fermion zero mode in the case of one instanton localised in Z_h . One can see that in regions around other instanton ($z \sim Z_j$, with $j \neq h$), the solution found above is of order $\frac{\rho_h \rho_j}{(Z_h - Z_j)^2} \ll 1$. So in this approximation there is one fermionic zero mode localised around each instanton. One has to note that the suppression of points distant from every instanton positions is larger than that obtained around $Z_{j \neq k}$. On these points we have low peak, suppressed with respect to that on Z_h , but larger with respect to the value of the single instanton profile at that point.

Vector Zero Modes

The vector zero modes in the adjoint representation of $SU(2)$ are those variations of A_μ that leave it a solution of the (anti-)selfdual equation (and that are not gauge transformations). They are associated to the parameter that describe the solution.

In the ADHM construction it is given the expression of the zero modes:

$$\delta A_\mu = -v^\dagger(\delta\alpha p\sigma_\mu b^\dagger - b\bar{\sigma}_\mu p\delta\alpha^\dagger)v \quad (\text{A.24})$$

Consider again the 't Hooft solution. There are $5k$ zero modes: $4k$ associated with changing positions of each instanton, and k with changing their sizes.

As an example, we give the expression for the zero mode relative to the translation of Z_j , by the vector Φ :

$$\begin{aligned} \delta A_\mu^\Phi &= \Phi_\nu \left[1 + \sum_{\ell=1}^k \frac{\rho_\ell^2}{(z - Z_\ell)^2} \right]^{-2} \frac{\rho_j^2}{(z - Z_j)^4} (\mathbf{z} - \mathbf{Z}_j)^\dagger \sigma_{\mu\nu} \times \\ &\times \left\{ \frac{(\mathbf{z} - \mathbf{Z}_j)}{(z - Z_j)^2} \left[1 + \sum_{i \neq j} \frac{\rho_i^2}{(z - Z_i)^2} \right] - \sum_{i \neq j} \frac{\rho_i^2}{(z - Z_i)^4} (\mathbf{z} - \mathbf{Z}_i) \right\} \end{aligned} \quad (\text{A.25})$$

One can see that in the limit (A.21) it becomes the zero mode of the single instanton solution localised on Z_j :

$$\delta A_\mu^\Phi \sim \Phi_\nu \frac{\rho_j^2}{(z - Z_j)^2} \frac{(\mathbf{z} - \mathbf{Z}_j)^\dagger \sigma_{\mu\nu} (\mathbf{z} - \mathbf{Z}_j)}{(\rho_j^2 + (z - Z_j)^2)^2} \quad (\text{A.26})$$

B Warping Effects on the Dirac Operator

We want to find the spin-connection relative to the metric:

$$\begin{aligned}
ds^2 &= f(r)^{-1/2} \tilde{g}_{(3,1)\mu\nu} dx^\mu dx^\nu + f(r)^{1/2} \tilde{g}_{(4)\alpha\beta} dz^\alpha dz^\beta \\
&= f(r)^{-1/2} \eta_{mn} \tilde{e}^m \tilde{e}^n + f(r)^{1/2} \delta_{ab} \tilde{e}^a \tilde{e}^b \\
&= \eta_{mn} e^m e^n + \delta_{ab} e^a e^b = \eta_{HK} e^H e^K
\end{aligned} \tag{B.1}$$

The corresponding 8-bein is given then by $e^m = f(r)^{-1/4} \tilde{e}^m$ and $e^a = f(r)^{1/4} \tilde{e}^a$. r is the radial coordinate in the 4-dimensional space spanned by the coordinates z^α .

The spin connection is given by:

$$\begin{aligned}
\omega_{\Pi}^{HK} &= \frac{1}{2} e^{\Lambda H} (\partial_{\Pi} e_{\Lambda}^K - \partial_{\Lambda} e_{\Pi}^K) - [H \leftrightarrow K] \\
&\quad - \frac{1}{2} e^{\Xi H} e^{\Upsilon K} (\partial_{\Xi} e_{\Upsilon Q} - \partial_{\Upsilon} e_{\Xi Q}) e_{\Pi}^Q
\end{aligned} \tag{B.2}$$

Using this formula, one obtains:

$$\begin{aligned}
\omega_{\mu}^{ab} &= \tilde{\omega}_{\mu}^{ab} \\
\omega_{\mu}^{an} &= \tilde{\omega}_{\mu}^{an} + \frac{f'}{4f^{3/2}} \tilde{e}_{\mu}^n \tilde{e}^{ra} \\
\omega_{\hat{\beta}}^{\hat{a}b} &= \tilde{\omega}_{\hat{\beta}}^{\hat{a}b} \\
\omega_{\hat{\beta}}^{\hat{a}R} &= \tilde{\omega}_{\hat{\beta}}^{\hat{a}R} + \frac{f'}{4f} \tilde{e}_{\hat{\beta}}^{\hat{a}} \\
\omega_r^{\hat{a}R} &= \tilde{\omega}_r^{\hat{a}R}
\end{aligned} \tag{B.3}$$

where $\tilde{\omega}$ is the spin connection associated to \tilde{g} and the coordinates x^α are split in the radial coordinate r and in the other three coordinates $x^{\hat{\alpha}}$ (and also $a = R, \hat{a}$).

The Dirac operator is given by

$$\mathcal{D}_8 = e_{\Pi}^K \Gamma^K (\partial_{\Pi} + \omega_{\Pi}^{HQ} \frac{1}{4} \Gamma_H \Gamma_Q + A_{\Pi}) \tag{B.4}$$

In the setup we are considering ($A_{\mu} = 0$ and ω given above), it is equal to

$$\begin{aligned}
\mathcal{D}_8 &= f^{1/4} \tilde{e}_m^{\mu} \Gamma^m (\partial_{\mu} + \omega_{\mu}^{HQ} \frac{1}{4} \Gamma_H \Gamma_Q) + f^{-1/4} \tilde{e}_a^{\alpha} \Gamma^a (\partial_{\alpha} + \omega_{\alpha}^{HQ} \frac{1}{4} \Gamma_H \Gamma_Q + A_{\alpha}) \\
&= f^{1/4} \tilde{e}_m^{\mu} \Gamma^m ((\tilde{D}_{3,1})_{\mu} + \delta \omega_{\mu}^{HQ} \frac{1}{4} \Gamma_H \Gamma_Q) + f^{-1/4} \tilde{e}_a^{\alpha} \Gamma^a ((\tilde{D}_4)_{\alpha} + \delta \omega_{\alpha}^{HQ} \frac{1}{4} \Gamma_H \Gamma_Q)
\end{aligned}$$

Where $\delta\omega = \omega - \tilde{\omega}$ can be read off above. In particular

$$\tilde{e}_m^\mu \Gamma^m \delta\omega_\mu^{HQ} \frac{1}{4} \Gamma_H \Gamma_Q = \frac{3}{8} \frac{f'}{f^{3/2}} \Gamma_r \quad (\text{B.5})$$

$$\tilde{e}_a^\alpha \Gamma^a \delta\omega_\alpha^{HQ} \frac{1}{4} \Gamma_H \Gamma_Q = -\frac{f'}{2f} \Gamma_r \quad (\text{B.6})$$

Putting all together one gets:

$$\begin{aligned} \mathcal{D}_8 &= f^{1/4} (\tilde{\mathcal{D}}_{3,1} \otimes \mathbf{1} + \frac{3}{8} \frac{f'}{f^{3/2}} \gamma^{(4)} \otimes \gamma_r) + f^{-1/4} (\gamma^{(4)} \otimes \mathcal{D}_4 - \frac{f'}{2f} \gamma^{(4)} \otimes \gamma_r) \\ &= f^{1/4} \tilde{\mathcal{D}}_{3,1} \otimes \mathbf{1} + f^{-1/4} \gamma^{(4)} \otimes \tilde{\mathcal{D}}_4 - \frac{1}{8f^{1/4}} \frac{f'}{f} \gamma^{(4)} \otimes \gamma_r \end{aligned} \quad (\text{B.7})$$

Splitting the 8-dimensional spinor as $\Psi = \sum_k \chi_k(x) \otimes \psi_k(y)$, we see that the zero modes of $\tilde{\mathcal{D}}_{3,1}$ are associated to the zero modes of the operator $\hat{\mathcal{D}}_4 = \tilde{\mathcal{D}}_4 - \frac{f'}{8f} \gamma_r$. If ψ_0 is a zero mode of $\tilde{\mathcal{D}}_4$, then $\psi = f^{1/8} \psi_0$ is a zero mode of $\hat{\mathcal{D}}_4$, since:

$$\tilde{\mathcal{D}}_4(f^{1/8} \psi_0) = \gamma_r (\partial_r f^{1/8}) \psi_0 = \frac{f'}{8f} \gamma_r (f^{1/8} \psi_0). \quad (\text{B.8})$$

References

- [1] L. Randall and R. Sundrum, “A large mass hierarchy from a small extra dimension,” *Phys. Rev. Lett.* **83** (1999) 3370 [arXiv:hep-ph/9905221]. “An alternative to compactification,” *Phys. Rev. Lett.* **83** (1999) 4690 [arXiv:hep-th/9906064].
- [2] H. Davoudiasl, J. L. Hewett and T. G. Rizzo, “Bulk gauge fields in the Randall-Sundrum model,” *Phys. Lett. B* **473** (2000) 43 [arXiv:hep-ph/9911262].
- [3] S. Chang, J. Hisano, H. Nakano, N. Okada and M. Yamaguchi, “Bulk standard model in the Randall-Sundrum background,” *Phys. Rev. D* **62** (2000) 084025 [arXiv:hep-ph/9912498].
- [4] T. Gherghetta and A. Pomarol, “Bulk fields and supersymmetry in a slice of AdS,” *Nucl. Phys. B* **586**, 141 (2000) [arXiv:hep-ph/0003129].
- [5] Y. Grossman and M. Neubert, “Neutrino masses and mixings in non-factorizable geometry,” *Phys. Lett. B* **474**, 361 (2000) [arXiv:hep-ph/9912408].
- [6] S. J. Huber and Q. Shafi, “Fermion masses, mixings and proton decay in a Randall-Sundrum model,” *Phys. Lett. B* **498** (2001) 256 [arXiv:hep-ph/0010195].
- [7] K. Agashe, A. Delgado, M. J. May and R. Sundrum, “RS1, custodial isospin and precision tests,” *JHEP* **0308** (2003) 050 [arXiv:hep-ph/0308036].
- [8] K. Agashe, R. Contino and A. Pomarol, “The minimal composite Higgs model,” *Nucl. Phys. B* **719** (2005) 165 [arXiv:hep-ph/0412089].
- [9] S. J. Huber and Q. Shafi, “Higgs mechanism and bulk gauge boson masses in the Randall-Sundrum model,” *Phys. Rev. D* **63** (2001) 045010 [arXiv:hep-ph/0005286].
- [10] C. Csaki, J. Erlich and J. Terning, “The effective Lagrangian in the Randall-Sundrum model and electroweak physics,” *Phys. Rev. D* **66** (2002) 064021 [arXiv:hep-ph/0203034].
- [11] J. L. Hewett, F. J. Petriello and T. G. Rizzo, “Precision measurements and fermion geography in the Randall-Sundrum model revisited,” *JHEP* **0209** (2002) 030 [arXiv:hep-ph/0203091].

- [12] A. Pomarol, “Grand unified theories without the desert,” *Phys. Rev. Lett.* **85** (2000) 4004 [arXiv:hep-ph/0005293].
- [13] L. Randall and M. D. Schwartz, “Quantum field theory and unification in AdS5,” *JHEP* **0111** (2001) 003 [arXiv:hep-th/0108114]; “Unification and the hierarchy from AdS5,” *Phys. Rev. Lett.* **88** (2002) 081801 [arXiv:hep-th/0108115].
- [14] W. D. Goldberger and I. Z. Rothstein, “High energy field theory in truncated AdS backgrounds,” *Phys. Rev. Lett.* **89** (2002) 131601 [arXiv:hep-th/0204160]; “Effective field theory and unification in AdS backgrounds,” *Phys. Rev. D* **68** (2003) 125011 [arXiv:hep-th/0208060].
- [15] K. Agashe, A. Delgado and R. Sundrum, “Gauge coupling renormalization in RS1,” *Nucl. Phys. B* **643** (2002) 172 [arXiv:hep-ph/0206099].
- [16] R. Contino, P. Creminelli and E. Trincherini, “Holographic evolution of gauge couplings,” *JHEP* **0210** (2002) 029 [arXiv:hep-th/0208002].
- [17] W. D. Goldberger, Y. Nomura and D. R. Smith, “Warped supersymmetric grand unification,” *Phys. Rev. D* **67** (2003) 075021 [arXiv:hep-ph/0209158].
- [18] K. Agashe and G. Servant, “Baryon number in warped GUTs: Model building and (dark matter related) phenomenology,” *JCAP* **0502** (2005) 002 [arXiv:hep-ph/0411254].
- [19] T. Gherghetta, “Warped models and holography,” arXiv:hep-ph/0601213.
- [20] N. Arkani-Hamed, M. Porrati and L. Randall, “Holography and phenomenology,” *JHEP* **0108** (2001) 017 [arXiv:hep-th/0012148].
R. Rattazzi and A. Zaffaroni, “Comments on the holographic picture of the Randall-Sundrum model,” *JHEP* **0104**, 021 (2001) [arXiv:hep-th/0012248].
- [21] S. B. Giddings, S. Kachru and J. Polchinski, “Hierarchies from fluxes in string compactifications,” *Phys. Rev. D* **66**, 106006 (2002) [arXiv:hep-th/0105097].
- [22] H. L. Verlinde, “Holography and compactification,” *Nucl. Phys. B* **580**, 264 (2000) [arXiv:hep-th/9906182].

- [23] J. F. G. Cascales, M. P. Garcia del Moral, F. Quevedo and A. M. Uranga, “Realistic D-brane models on warped throats: Fluxes, hierarchies and moduli stabilization,” JHEP **0402**, 031 (2004) [arXiv:hep-th/0312051].
- [24] T. Gherghetta and J. Giedt, “Bulk fields in AdS(5) from probe D7 branes,” Phys. Rev. D **74** (2006) 066007 [arXiv:hep-th/0605212].
- [25] H. Osborn, “Solutions Of The Dirac Equation For General Instanton Solutions,” Nucl. Phys. B **140** (1978) 45.
- [26] M. R. Douglas, “Branes within branes,” arXiv:hep-th/9512077.
- [27] J. M. Maldacena, “The large N limit of superconformal field theories and supergravity,” Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)] [arXiv:hep-th/9711200].
- [28] *Work in Progress*
- [29] C. P. Burgess, P. G. Camara, S. P. de Alwis, S. B. Giddings, A. Maharana, F. Quevedo and K. Suruliz, “Warped supersymmetry breaking,” arXiv:hep-th/0610255.
- [30] M. F. Atiyah, N. J. Hitchin, V. G. Drinfeld and Yu. I. Manin, “Construction of instantons,” Phys. Lett. A **65** (1978) 185.
- [31] N. Dorey, T. J. Hollowood, V. V. Khoze and M. P. Mattis, “The calculus of many instantons,” Phys. Rept. **371** (2002) 231 [arXiv:hep-th/0206063].
- [32] G. 't Hooft, “Computation of the quantum effects due to a four-dimensional pseudoparticle,” Phys. Rev. D **14**, 3432 (1976) [Erratum-ibid. D **18**, 2199 (1978)].