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Charged Lepton Decays $l_i \to l_j + \gamma$, Leptogenesis CP-Violating Parameters and Majorana Phases

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Abstract

We analyse the dependence of the rates of the LFV charged lepton decays $\mu \to e + \gamma$, $\tau \to e+\gamma$, $\tau \to \mu+\gamma$ ($l_i \to l_j+\gamma$) and their ratios, predicted in the class of SUSY theories with see-saw mechanism of ν-mass generation and soft SUSY breaking with universal boundary conditions at the GUT scale, on the Majorana CP-violation phases in the PMNS neutrino mixing matrix and the "leptogenesis" CP-violating (CPV) parameters. The case of quasi-degenerate in mass heavy Majorana neutrinos is considered. The analysis is performed for normal hierarchical (NH), inverted hierarchical (IH) and quasi-degenerate (QD) light neutrino mass spectra. We show, in particular, that for NH and IH ν-mass spectrum and negligible lightest neutrino mass, all three $l_i \to l_j + \gamma$ decay branching ratios, $BR(l_i \to l_j + \gamma)$, depend on one Majorana phase, one leptogenesis CPV parameter and on the 3-neutrino oscillation parameters; if the CHOOZ mixing angle θ_{13} is sufficiently large, they depend on the Dirac CPV phase in the PMNS matrix. The "double ratios" R(21/31) $\sim BR(\mu \to e + \gamma)/BR(\tau \to e + \gamma)$ and R(21/32) $\sim BR(\mu \to e + \gamma)/BR(\tau \to \mu + \gamma)$ are determined by these parameters. The same Majorana phase enters into the NH and IH expressions for the effective Majorana mass in neutrinoless double beta decay, $<m>$.

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1 Introduction

The experiments with solar, atmospheric, reactor and accelerator neutrinos [1–5] have provided during the last several years compelling evidence for the existence of non-trivial 3-neutrino mixing in the weak charged-lepton current (see, e.g., [6]):

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}}, \quad l = e, \mu, \tau, \tag{1}$$

where ν_{lL} are the flavour neutrino fields, $\nu_{j\mathrm{L}}$ is the field of neutrino ν_j having a mass m_j and U is the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) mixing matrix [7], $U \equiv U_{\mathrm{PMNS}}$. The existing data, including the data from the ^{3}H β-decay experiments [8] imply that the massive neutrinos ν_j are significantly lighter than the charged leptons and quarks: $m_j < 2.3$ eV (95% C.L.) [1].

The existence of the flavour neutrino mixing, eq. (1), implies that the individual lepton charges, L_l, $l = e, \mu, \tau$, are not conserved (see, *e.g.*, [11]), and processes like $\mu^- \to e^- + \gamma$, $\mu^- \to e^- + e^+ + e^-$, $\tau^- \to e^- + \gamma$, $\tau^- \to \mu^- + \gamma$, $\mu^- + (A,Z) \to e^- + (A,Z)$, *etc.* should take place. Stringent experimental upper limits on the branching ratios and relative cross-sections of the indicated $|\Delta L_l| = 1$ decays and reactions have been obtained [12–14] (90% C.L.):

$$\begin{aligned} \mathrm{BR}(\mu \to e + \gamma) < 1.2 \times 10^{-11}, \quad \mathrm{BR}(\mu \to 3e) < 1.2 \times 10^{-12}\ , \\ \mathrm{BR}(\tau \to \mu + \gamma) < 6.8 \times 10^{-8}\ , \quad \mathrm{R}(\mu^- + \mathrm{Ti} \to e^- + \mathrm{Ti}) < 4.3 \times 10^{-12}. \end{aligned} \tag{2}$$

Future experiments with increased sensitivity can reduce the current bounds on $\mathrm{BR}(\mu \to e + \gamma)$, $\mathrm{BR}(\tau \to \mu + \gamma)$ and on $\mathrm{R}(\mu^- + (A,Z) \to e^- + (A,Z))$ by a few orders of magnitude (see, *e.g.*, [15]). In the experiment MEG under preparation at PSI [16] it is planned to reach a sensitivity to

$$\mathrm{BR}(\mu \to e + \gamma) \sim (10^{-13} - 10^{-14})\,. \tag{3}$$

In the minimal extension of the Standard Theory with massive neutrinos and neutrino mixing, the rates and cross sections of the LFV processes are suppressed by the factor [17] (see also [18]) $(m_j/M_W)^4 < 6.7 \times 10^{-43}$, M_W being the $W^\pm$ mass, which renders them unobservable. It was shown in [19] that in SUSY theories with see-saw mechanism of neutrino mass generation [2] [21] and soft SUSY breaking with universal boundary conditions at a scale M_X above the right-handed (RH) Majorana neutrino mass scale M_R, $M_X > M_R$ the rates and cross sections of the LFV processes can be strongly enhanced and can be within the sensitivity of presently operating and future planned experiments (see also, e.g., [22–31]). As is well-known, the see-saw mechanism of neutrino mass generation [21], provides a very attractive explanation of the smallness of the neutrino masses and - through the leptogenesis theory [32], of the observed baryon asymmetry of the Universe.

One of the basic ingredients of the see-saw mechanism is the matrix of neutrino Yukawa couplings, $\mathbf{Y}_\nu$. Leptogenesis depends on $\mathbf{Y}_\nu$ as well. In the large class of SUSY models with

[1] More stringent upper limit on m_j follows from the constraints on the sum of neutrino masses obtained from cosmological/astrophysical observations, namely, the CMB data of the WMAP experiment combined with data from large scale structure surveys (2dFGRS, SDSS) [9]: $\sum_j m_j < (0.7-2.0)$ eV (95% C.L.), where we have included a conservative estimate of the uncertainty in the upper limit (see, *e.g.*, [10]).

[2] An integral part of the see-saw mechanism are the right-handed (heavy) Majorana neutrinos [20].

see-saw mechanism and SUSY breaking mediated by flavour-universal soft terms at a scale $M_X > M_R$ we will consider, the probabilities of LFV processes also depend strongly on $\mathbf{Y}_\nu$ (see, *e.g.*, [23,24]). The matrix $\mathbf{Y}_\nu$ can be expressed in terms of the light neutrino and heavy RH neutrino masses, the neutrino mixing matrix $U_{\rm PMNS}$, and an orthogonal matrix $\mathbf{R}$ [23]. Leptogenesis can take place only if $\mathbf{R}$ is complex. Obviously, $\mathbf{Y}_\nu$ depends on the Majorana CP-violation (CPV) phases in the PMNS matrix $U_{\rm PMNS}$ [33].

It was shown in [26] that if both the light and the heavy Majorana neutrino mass spectra are quasi-degenerate (QD), the rates of LFV decays $\mu \to e + \gamma$, $\tau \to e + \gamma$ and $\tau \to \mu + \gamma$, predicted in the class of SUSY theories of interest, can be strongly enhanced by the leptogenesis CP-violating (CPV) parameters in $\mathbf{R} \neq \mathbf{1}$, with respect to the rates predicted for $\mathbf{R} = \mathbf{1}$. The indicated LFV decay rates were also noticed in [26] to depend for $\mathbf{R} \neq \mathbf{1}$ on the Majorana CPV phases in $U_{\rm PMNS}$. This dependence was investigated recently in [30] by taking into account the effects of the phases in the renormalisation group (RG) running of the light ν-masses m_j and of the mixing angles in $U_{\rm PMNS}$. It was found [30] that the Majorana phases can affect significantly the predictions for the $\mu \to e + \gamma$ and $\tau \to e + \gamma$ decay rates.

In the present article we extend the analyses performed in [26,30] to the cases of normal hierarchical and inverted hierarchical light neutrino mass spectra. We investigate also in greater detail the case of QD spectrum. More specifically, working in the framework of the class of SUSY theories with see-saw mechanism and soft SUSY breaking with flavour-universal boundary conditions at a scale $M_X > M_R$, we study in detail the dependence of the rates of charged lepton flavour violating radiative decays $\mu \to e + \gamma$, $\tau \to e + \gamma$ and $\tau \to \mu + \gamma$, on the Majorana CPV phases in $U_{\rm PMNS}$ and on the leptogenesis CPV parameters in the matrix $\mathbf{R}$. The case of quasi-degenerate in mass heavy RH Majorana neutrinos is considered. Our analysis is performed under the condition of negligible RG effects for the light neutrino masses m_j and the mixing angles and CP-violation phases in $U_{\rm PMNS}$. The RG effects in question (see, e.g., [30,34] and the references quoted therein) are negligibly small in the class of SUSY theories we are considering in the case of hierarchical (normal or inverted) ν_j mass spectrum. The same is valid for QD ν_j mass spectrum provided the SUSY parameter $\tan\beta$ is relatively small, $\tan\beta < 10$, $\tan\beta$ being the ratio of the vacuum expectation values of the up- and down- type Higgs doublet fields in SUSY extensions of the Standard Theory. For the three types of light neutrino mass spectrum, we investigate also the predictions for the ratios of the rates of $\mu \to e+\gamma$ and $\tau \to e+\gamma$, and of $\mu \to e+\gamma$ and $\tau \to \mu+\gamma$, decays. In a large region of the relevant SUSY parameter space these two ratios are independent of the SUSY parameters and are determined completely by the neutrino mixing angles, Majorana and Dirac CPV phases, leptogenesis CPV parameter(s) and, depending on the type of the neutrino mass spectrum - hierarchical or quasi-degenerate, by the neutrino mass squared differences Δm^2_{21} and Δm^2_{31} or the absolute neutrino mass. A study of the predictions for the two LFV decay rate ratios was performed recently in ref. [35]. In [35], however, only the case of zero leptogenesis CPV parameters and zero Majorana CPV phases in $U_{\rm PMNS}$ was investigated.

2 Neutrino Mixing Parameters from Neutrino Oscillation Data

We will use the standard parametrisation of the PMNS matrix $U_{\rm PMNS}$ (see, *e.g.*, [36]):

$$U_{\rm PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} {\rm diag}(1, e^{i\frac{\alpha}{2}}, e^{i\frac{\beta_M}{2}})\,, \tag{4}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, the angles $\theta_{ij} = [0, \pi/2]$, $\delta = [0, 2\pi]$ is the Dirac CP-violating phase and α and β_M are two Majorana CP-violation phases [33, 37]. One can identify the neutrino mass squared difference responsible for solar neutrino oscillations, $\Delta m^2_\odot$, with $\Delta m^2_{21} \equiv m^2_2 - m^2_1$, $\Delta m^2_\odot = \Delta m^2_{21} > 0$. The neutrino mass squared difference driving the dominant $\nu_\mu \to \nu_\tau$ ($\bar{\nu}_\mu \to \bar{\nu}_\tau$) oscillations of atmospheric ν_μ ($\bar{\nu}_\mu$) is then given by $|\Delta m^2_{\rm A}| = |\Delta m^2_{31}| \cong |\Delta m^2_{32}| \gg \Delta m^2_{21}$. The corresponding solar and atmospheric neutrino mixing angles, $\theta_\odot$ and $\theta_{\rm A}$, coincide with θ_{12} and θ_{23}, respectively. The angle θ_{13} is limited by the data from the CHOOZ and Palo Verde experiments [38].

The existing neutrino oscillation data allow us to determine Δm^2_{21}, $|\Delta m^2_{31}|$, $\sin^2\theta_{12}$ and $\sin^2 2\theta_{23}$ with a relatively good precision and to obtain rather stringent limits on $\sin^2\theta_{13}$ (see, *e.g.*, [39, 40]). The best fit values and the 95% C.L. allowed ranges of Δm^2_{21}, $\sin^2\theta_{12}$, $|\Delta m^2_{31}|$ and $\sin^2 2\theta_{23}$ read [3]:

$$\begin{aligned} \Delta m^2_{21} = 8.0 \times 10^{-5}~{\rm eV}^2, \quad \sin^2\theta_{21} = 0.31~, \\ \Delta m^2_{21} = (7.3 - 8.5) \times 10^{-5}~{\rm eV}^2, \quad \sin^2\theta_{12} = (0.26 - 0.36)~, \end{aligned} \tag{5}$$

$$\begin{aligned} |\Delta m^2_{31}| = 2.2 \times 10^{-3}~{\rm eV}^2~, \quad \sin^2 2\theta_{23} = 1.0~, \\ |\Delta m^2_{31}| = (1.7 - 2.9) \times 10^{-3}~{\rm eV}^2~, \quad \sin^2 2\theta_{23} \geq 0.90. \end{aligned} \tag{6}$$

A combined [4] 3-ν oscillation analysis of the solar neutrino, KL and CHOOZ data gives [39]

$$\sin^2\theta_{13} < 0.027~(0.044), \qquad \text{at } 95\%~(99.73\%)~\text{C.L.} \tag{7}$$

The neutrino oscillation parameters discussed above can (and very likely will) be measured with much higher accuracy in the future (see, *e.g.*, [6]).

The sign of $\Delta m^2_{\rm A} = \Delta m^2_{31}$, as it is well known, cannot be determined from the present (SK atmospheric neutrino and K2K) data. The two possibilities, $\Delta m^2_{31(32)} > 0$ or $\Delta m^2_{31(32)} < 0$ correspond to two different types of ν-mass spectrum:
– *with normal ordering (hierarchy)* $m_1 < m_2 < m_3$, $\Delta m^2_{\rm A} = \Delta m^2_{31} > 0$, and
– *with inverted ordering (hierarchy)* $m_3 < m_1 < m_2$, $\Delta m^2_{\rm A} = \Delta m^2_{32} < 0$.
Depending on the sign of $\Delta m^2_{\rm A}$, ${\rm sgn}(\Delta m^2_{\rm A})$, and the value of the lightest neutrino mass, ${\rm min}(m_j)$, the ν-mass spectrum can be

[3] The data imply, in particular, that maximal solar neutrino mixing is ruled out at $\sim 6\sigma$; at 95% C.L. one finds $\cos 2\theta_\odot \geq 0.26$ [39], which has important implications [41].

[4] Using the recently announced (but still unpublished) data from the MINOS experiment [42] in the analysis leads to somewhat different best fit value and 95% allowed range of $|\Delta m^2_{31}|$ [43]: $|\Delta m^2_{31}| = 2.5 \times 10^{-3}$ eV^2 and $|\Delta m^2_{31}| = (2.2 - 2.9) \times 10^{-3}$ eV^2.

– *Normal Hierarchical*: $m_1 \ll m_2 \ll m_3$, $m_2 \cong (\Delta m^2_\odot)^{\frac{1}{2}} \sim 0.009$ eV, $m_3 \cong |\Delta m^2_{\rm A}|^{\frac{1}{2}} \sim 0.05$ eV;
– *Inverted Hierarchical*: $m_3 \ll m_1 < m_2$, with $m_{1,2} \cong |\Delta m^2_{\rm A}|^{\frac{1}{2}} \sim 0.05$ eV;
– *Quasi-Degenerate (QD)*: $m_1 \cong m_2 \cong m_3 \cong m$, $m_j^2 \gg |\Delta m^2_{\rm A}|$, $m \gtrsim 0.10$ eV.

The sign of $\Delta m^2_{31} \cong \Delta m^2_{32}$, which drives the dominant atmospheric neutrino oscillations, can be determined by studying oscillations of neutrinos and antineutrinos, say, $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$, in which matter effects are sufficiently large. This can be done, e.g., in long-baseline ν-oscillation experiments (see, e.g., [44]). Information about sgn(Δm^2_{31}) can be obtained also in atmospheric neutrino experiments by studying the oscillations of the atmospheric ν_μ and $\bar{\nu}_\mu$ which traverse the Earth [45].

As is well-known, the theories employing the see-saw mechanism of neutrino mass generation [21] of interest for our discussion, predict the massive neutrinos ν_j to be Majorana particles. Determining the nature of massive neutrinos is one of the most formidable and pressing problems in today's neutrino physics (see, *e.g.*, [6, 46]). If it is established that the massive neutrinos ν_j are indeed Majorana fermions, getting information about the Majorana CP-violation phases in $U_{\rm PMNS}$, would be a very difficult problem. The oscillations of flavour neutrinos, $\nu_l \to \nu_{l'}$ and $\bar{\nu}_l \to \bar{\nu}_{l'}$, $l, l' = e, \mu, \tau$, are insensitive to the Majorana CP-violation phases α and β_M [33, 47]. The only feasible experiments that at present have the potential of establishing the Majorana nature of light neutrinos ν_j and of providing information on the Majorana CP-violation phases in $U_{\rm PMNS}$ are the experiments searching for the neutrinoless double beta ($(\beta\beta)_{0\nu}$-) decay, $(A, Z) \to (A, Z+2) + e^- + e^-$ (see, *e.g.*, [11, 46, 48]). The $(\beta\beta)_{0\nu}$-decay effective Majorana mass, $<\!m\!>$ (see, e.g., [11]), which contains all the dependence of the $(\beta\beta)_{0\nu}$-decay amplitude on the neutrino mixing parameters, is given by the following expressions for the normal hierarchical (NH), inverted hierarchical (IH) and quasi-degenerate (QD) neutrino mass spectra (see, e.g., [48]):

$$|\!<\!m\!>\!| \;\cong\; \left|\sqrt{\Delta m^2_{21}}\,\sin^2\theta_{12}e^{i\alpha} + \sqrt{\Delta m^2_{31}}\,\sin^2\theta_{13}e^{i\beta_M}\right| \,,\; m_1 \ll m_2 \ll m_3 \;({\rm NH}), \tag{8}$$

$$|\!<\!m\!>\!| \;\cong\; \sqrt{\Delta m^2_{13}}\left|\cos^2\theta_{12} + e^{i\alpha}\,\sin^2\theta_{12}\right| \,,\quad m_3 \ll m_1 < m_2 \;({\rm IH}), \tag{9}$$

$$|\!<\!m\!>\!| \;\cong\; m\left|\cos^2\theta_{12} + e^{i\alpha}\,\sin^2\theta_{12}\right| \,,\quad m_{1,2,3} \cong m \gtrsim 0.10\ {\rm eV}\;\;({\rm QD}). \tag{10}$$

Obviously, $|\!<\!m\!>\!|$ depends strongly on the Majorana CP-violation phase(s) [5]; the CP-conserving values of $(\alpha - \beta_M) = 0, \pm\pi$ $(\alpha = 0, \pm\pi)$ [49], in particular, determine the range of possible values of $|\!<\!m\!>\!|$ in the case of NH (IH, QD) spectrum. If the $(\beta\beta)_{0\nu}$-decay is observed, the measurement of the $(\beta\beta)_{0\nu}$-decay half-life combined with information on the absolute scale of neutrino masses (or on min(m_j)), might allow to significantly constrain the Majorana phase α [36, 50, 51], for instance.

[5] We assume that the fields of the Majorana neutrinos ν_j satisfy the Majorana conditions: $C(\bar{\nu}_{1,2})^T = \nu_{1,2}$, and $C(\bar{\nu}_3)^T = e^{-i2\delta}\nu_3$, where C is the charge conjugation matrix. With the parametrisation we are employing for $U_{\rm PMNS}$, eq. (4), the effective Majorana mass $|\!<\!m\!>\!|$ does not depend on the Dirac CP-violation phase δ as a consequence of the presence of the phase factor $e^{-i2\delta}$ in the Majorana condition for the field ν_3.

3 The See-Saw Mechanism, Neutrino Yukawa Couplings, and LFV Decays $l_i \to l_j + \gamma$

In the minimal supersymmetric standard model with RH neutrinos and see-saw mechanism of neutrino mass generation (MSSMRN) we consider it is always possible to choose a basis in which both the matrix of charged lepton Yukawa couplings, $\mathbf{Y}_{\rm E}$, and the Majorana mass matrix of the heavy RH neutrinos, $\mathbf{M}_{\rm N}$, are real and diagonal. We will work in that basis and will denote by $\mathbf{D}_{\rm N}$ the corresponding diagonal RH neutrino mass matrix, $\mathbf{D}_{\rm N} = {\rm diag}(M_1, M_2, M_3)$, with $M_j > 0$. We will consider in what follows the case of QD heavy Majorana neutrinos: $M_1 \cong M_2 \cong M_3 = M_R$. The heavy Majorana neutrino mass M_R will standardly be assumed to be smaller than the GUT scale $M_{\rm GUT} \simeq 2 \times 10^{16}$ GeV.

In the class of theories of interest, the branching ratio of the $l_i \to l_j + \gamma$ decay has the following form (in the "mass insertion" and leading-log approximations, see, e.g., [22,24,28]):

$$\mathrm{BR}(l_i \to l_j \gamma) \cong \frac{\Gamma(l_i \to e\nu\bar{\nu})}{\Gamma_{\rm total}(l_i)} \frac{\alpha_{\rm em}^3}{G_F^2 m_S^8} \left|\frac{(3+a_0^2)m_0^2}{8\pi^2}\right|^2 \left|\sum_k (Y_\nu^\dagger)^{ik} \, \ln\frac{M_X}{M_k} \, Y_\nu^{kj}\right|^2 \tan^2\beta \,, \qquad (11)$$

where $i \neq j = 1,2,3$, $l_1, l_2, l_3 \equiv e, \mu, \tau$, m_0 and $A_0 = a_0 m_0$ are the universal SUSY breaking scalar masses and trilinear scalar couplings at $M_X > M_R$, m_S represents SUSY particle mass (see further), $\tan\beta$ is the ratio of the vacuum expectation values of up-type and down-type Higgs fields and $\mathbf{Y}_\nu = \mathbf{Y}_\nu(M_R)$ is the matrix of neutrino Yukawa couplings evaluated at M_R. The matrix $\mathbf{Y}_\nu$ can be parametrised as [23]

$$\mathbf{Y}_\nu(M_R) = \frac{1}{v_u}\sqrt{\mathbf{D}_N}\; \mathbf{R}\; \sqrt{\mathbf{D}_\nu}\; \mathbf{U}^\dagger \cong \frac{1}{v_u}\sqrt{M_R}\; \mathbf{R}\; \sqrt{\mathbf{D}_\nu}\; \mathbf{U}^\dagger \,. \qquad (12)$$

Here $v_u = v\sin\beta$, where $v = 174$ GeV, $\mathbf{R}$ is a complex orthogonal matrix [6] $\mathbf{R}^T\mathbf{R} = \mathbf{1}$, $\mathbf{D}_\nu = {\rm diag}(m_1, m_2, m_3)$, $m_{1,2,3} > 0$ being the light neutrino masses [7] and $\mathbf{U}$ is the PMNS matrix.

In what follows we will consider the case when the RG running of m_j and of the parameters in $U_{\rm PMNS}$ from approximately $M_Z \sim 100$ GeV, where they are measured, to M_R is relatively small and can be neglected. This possibility is realised in the class of theories under discussion for sufficiently small values of $\tan\beta$ and/or of the lightest neutrino mass $\min(m_j)$, e.g., for $\tan\beta < 10$ and/or $\min(m_j) \lesssim 0.05$ eV (see, e.g., [30, 34]). Under the indicated condition, $\mathbf{D}_\nu$ and $\mathbf{U}$ in eq. (12) should be taken at the scale $\sim M_Z$, at which the neutrino mixing parameters are measured.

It was shown in [28] that in a large region of the relevant soft SUSY breaking parameter space, the expression

$$m_S^8 \simeq 0.5\; m_0^2\; m_{1/2}^2\; (m_0^2 + 0.6\; m_{1/2}^2)^2 \,, \qquad (13)$$

[6] Equation (12) represents the so-called "orthogonal" parametrisation of $\mathbf{Y}_\nu$. In certain cases it is more convenient to use the "bi-unitary" parametrisation [27] $\mathbf{Y}_\nu = \mathbf{U}_R^\dagger \mathbf{Y}_\nu^{\rm diag}\, \mathbf{U}_L$, where $\mathbf{U}_{\rm L,R}$ are unitary matrices and $\mathbf{Y}_\nu^{\rm diag}$ is a real diagonal matrix. The orthogonal parametrisation is better adapted for our analysis and we will employ it in what follows.

[7] To be more precise, we can have $\min(m_j) = 0$.

$m_{1/2}$ being the universal gaugino mass at M_X, gives an excellent approximation to the results obtained in a full renormalisation group analysis, i.e., without using the leading-log and the mass insertion approximations. For values of the soft SUSY breaking parameters implying SUSY particle masses in the range of few to several hundred GeV, say, $m_0 = m_{1/2} = 250$ GeV, $A_0 = a_0 m_0 = -100$ GeV, we have:

$$BR(l_i \to l_j \gamma) \cong 9.1 \times 10^{-10} \left|(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu)_{ij}\right|^2 \tan^2\beta \,, \tag{14}$$

where $L \cong \ln(M_X/M_R)$. Since $\tan^2\beta \gtrsim 10$, eq. (14) implies that if indeed the SUSY particle masses do not exceed several hundred GeV, the quantity $|(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu)_{21}|$ has to be relatively small. This is realised for, e.g., $M_R \lesssim 10^{12}$ GeV.

As follows from eqs. (11) and (14) and was widely discussed , in the case of soft SUSY breaking mediated by soft flavour-universal terms at $M_X > M_R$, the predicted rates of LFV processes such as $\mu \to e + \gamma$ decay are very sensitive to the off-diagonal elements of

$$\mathbf{Y}_\nu^\dagger(M_R)\mathbf{Y}_\nu(M_R) = \frac{1}{v_u^2}\,\mathbf{U}\sqrt{\mathbf{D}_\nu}\,\mathbf{R}^\dagger\,\mathbf{D}_N\,\mathbf{R}\,\sqrt{\mathbf{D}_\nu}\mathbf{U}^\dagger \cong \frac{M_R}{v_u^2}\,\mathbf{U}\sqrt{\mathbf{D}_\nu}\,\mathbf{R}^\dagger\,\mathbf{R}\,\sqrt{\mathbf{D}_\nu}\mathbf{U}^\dagger \,. \tag{15}$$

It is well-known that in the theories with see-saw mechanism, leptogenesis depends on [32]

$$\mathbf{Y}_\nu(M_R)\mathbf{Y}_\nu^\dagger(M_R) = \frac{1}{v_u^2}\,\sqrt{\mathbf{D}_\mathrm{N}}\,\mathbf{R}\,\mathbf{D}_\nu\,\mathbf{R}^\dagger\,\sqrt{\mathbf{D}_\mathrm{N}} \cong \frac{M_R}{v_u^2}\,\mathbf{R}\,\mathbf{D}_\nu\,\mathbf{R}^\dagger \,. \tag{16}$$

Successful leptogenesis can take place only if $\mathbf{R}$ is complex, so we will consider $(\mathbf{R})^* \neq \mathbf{R}$. In what follows we will use the parameterizations of $\mathbf{R}$ proposed in [26]:

$$\mathbf{R} = \mathbf{O}\, e^{i\mathbf{A}} \,. \tag{17}$$

Here $\mathbf{O}$ is a *real orthogonal* matrix

$$\mathbf{O} = \mathbf{O}_{12}\,\mathbf{O}_{13}\,\mathbf{O}_{23} \,, \tag{18}$$

where $\mathbf{O}_{ij}$ are 2×2 orthogonal matrices of rotations with real angles ρ_{ij}, and $\mathbf{A}$ is a *real antisymmetric* matrix, $(\mathbf{A})^T = -\mathbf{A}$,

$$A = \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix} \,, \tag{19}$$

a, b, c being real parameters. The following representation of $e^{i\mathbf{A}}$ proves useful [26]:

$$e^{i\mathbf{A}} = \mathbf{1} - \frac{\cosh r - 1}{r^2}\mathbf{A}^2 + i\frac{\sinh r}{r}\mathbf{A} \,, \tag{20}$$

with $r = \sqrt{a^2 + b^2 + c^2}$. The requirement of successful leptogenesis in the case of QD light and heavy Majorana neutrino mass spectra implies [26] that $abc \neq 0$ and that none of the parameters $|a|$, $|b|$ and $|c|$ can be exceedingly small: $|abc| \sim (10^{-6} - 10^{-4})$. One also finds from the condition that Yukawa couplings should have moduli which do not exceed ~ 1 that typically $r \lesssim 1$ [26].

The parametrisation given in eq. (17) is particularly convenient in the analysis of the case of QD heavy Majorana neutrinos. We will consider a range of values of the parameters a, b, c determined by $10^{-4} \lesssim |a|, |b|, |c| \lesssim 0.10$. Equations (11) and (15) imply that for QD heavy Majorana neutrinos we can set $\mathbf{O} = \mathbf{1}$ and use $\mathbf{R} = e^{i\mathbf{A}}$ in the calculation of $\mathrm{BR}(l_i \to l_j + \gamma)$ without loss of generality.

4 The LFV Decays $l_i \to l_j + \gamma$ and Majorana Phases

In the case under discussion $M_1 = M_2 = M_3 \equiv M_R$ and the matrix of neutrino Yukawa couplings has the form $\mathbf{Y}_\nu = \frac{\sqrt{M_R}}{v_u}\mathbf{O}e^{i\mathbf{A}}\sqrt{\mathbf{D}_\nu}\mathbf{U}^\dagger$. The off-diagonal elements of $\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu$ of interest do not depend on $\mathbf{O}$ and satisfy $(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)^*_{ij} = (\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)_{ji}$. To leading order in small quantities they are given by (see also [26, 30])

$$\begin{aligned}(Y_\nu^\dagger Y_\nu)_{12} &= \Delta_{21}c_{23}c_{12}s_{12} + \Delta_{31}s_{23}s_{13}e^{-i\delta} \\ &+ 2\frac{M_R}{v_u^2}i\Big[a\sqrt{m_1m_2}\left(c_{23}(c_{12}^2e^{-i\frac{\alpha}{2}} + s_{12}^2e^{i\frac{\alpha}{2}}) + 2i\ s_{13}c_{12}s_{12}s_{23}\ e^{-i\delta}\sin\frac{\alpha}{2}\right) \\ &\quad + b\sqrt{m_1m_3}s_{23}\left(c_{12}e^{-i\frac{\beta_M}{2}} - s_{13}s_{12}e^{i(\frac{\beta_M}{2}-\delta)}\right) \\ &\quad + c\sqrt{m_2m_3}\left(s_{23}s_{12}e^{i\frac{\alpha-\beta_M}{2}} - c_{23}s_{13}c_{12}e^{-i(\frac{\alpha-\beta_M}{2}+\delta)}\right) + \mathcal{O}(s_{13}^2)\Big] + \mathcal{O}(r^2, s_{13}^2)\ ,\end{aligned} \tag{21}$$

$$\begin{aligned}(Y_\nu^\dagger Y_\nu)_{13} &= -\Delta_{21}s_{23}c_{12}s_{12} + \Delta_{31}c_{23}s_{13}e^{-i\delta} \\ &+ 2\frac{M_R}{v_u^2}i\Big[a\sqrt{m_1m_2}\left(-s_{23}(c_{12}^2e^{-i\frac{\alpha}{2}} + s_{12}^2e^{i\frac{\alpha}{2}}) + 2i\ s_{13}c_{12}s_{12}c_{23}\ e^{-i\delta}\sin\frac{\alpha}{2}\right) \\ &\quad + b\sqrt{m_1m_3}\left(c_{12}c_{23}e^{-i\frac{\beta_M}{2}} - s_{13}s_{12}s_{23}e^{i(\frac{\beta_M}{2}-\delta)}\right) \\ &\quad + c\sqrt{m_2m_3}\left(s_{12}c_{23}e^{i\frac{\alpha-\beta_M}{2}} + s_{13}c_{12}s_{23}e^{-i(\frac{\alpha-\beta_M}{2}+\delta)}\right) + \mathcal{O}(s_{13}^2)\Big] + \mathcal{O}(r^2, s_{13}^2)\ ,\end{aligned} \tag{22}$$

$$\begin{aligned}(Y_\nu^\dagger Y_\nu)_{23} &= \Delta_{31}s_{23}c_{23} \\ &+ 2\frac{M_R}{v_u^2}i\Big[a\sqrt{m_1m_2}\Big(-2ic_{12}s_{12}c_{23}s_{23}\sin\frac{\alpha}{2} + s_{13}\left[c_{12}^2(s_{23}^2e^{-i(\frac{\alpha}{2}-\delta)} + c_{23}^2e^{i(\frac{\alpha}{2}-\delta)})\right. \\ &\quad \left. + s_{12}^2(s_{23}^2e^{i(\frac{\alpha}{2}+\delta)} + c_{23}^2e^{-i(\frac{\alpha}{2}+\delta)})\right]\Big) \\ &\quad + b\sqrt{m_1m_3}\left(-s_{12}(s_{23}^2e^{i\frac{\beta_M}{2}} + c_{23}^2e^{-i\frac{\beta_M}{2}}) + s_{13}c_{12}c_{23}s_{23}\ 2i\sin(\frac{\beta_M}{2}-\delta)\right) \\ &\quad + c\sqrt{m_2m_3}\left(c_{12}(c_{23}^2e^{i\frac{\alpha-\beta_M}{2}} + s_{23}^2e^{-i\frac{\alpha-\beta_M}{2}}) - s_{13}s_{12}c_{23}s_{23}\ 2i\sin(\frac{\alpha-\beta_M}{2}+\delta)\right) \\ &\quad + \mathcal{O}(s_{13}^2)\Big] + \mathcal{O}(r^2, s_{13}^2)\ ,\end{aligned} \tag{23}$$

where

$$\Delta_{ij} \equiv \frac{M_R}{v_u^2}(m_i - m_j) = \frac{M_R}{v_u^2}\ \frac{\Delta m_{ij}^2}{m_i + m_j}\ . \tag{24}$$

Equations (21)-(23) are valid for any of the possible types of light neutrino mass spectrum. The above results imply that in the absence of significant RG effects, the "double" ratios

$$\mathrm{R}(21/31) \equiv \frac{\mathrm{BR}(\mu\to e+\gamma)}{\mathrm{BR}(\tau\to e+\gamma)}\ \mathrm{BR}(\tau\to e\nu_\tau\bar{\nu}_e)\ ,\ \mathrm{R}(21/32) \equiv \frac{\mathrm{BR}(\mu\to e+\gamma)}{\mathrm{BR}(\tau\to \mu+\gamma)}\ \mathrm{BR}(\tau\to e\nu_\tau\bar{\nu}_e)\ , \tag{25}$$

depend in the region of validity of eqs. (11) and (13) in the relevant SUSY parameter space, *on the neutrino masses* m_j, *mixing angles* θ_{12}, θ_{23}, θ_{13} *and Majorana and Dirac CP-violation phases* α, β_M *and* δ *at* $\sim M_Z$, *as well as on the the "leptogenesis CP-violating (CPV) parameters"* a, b *and* c. The dependence of the Dirac phase δ can be significant only if the CHOOZ angle θ_{13} is sufficiently large. Although the general expressions for $(Y_\nu^\dagger Y_\nu)_{ij}$, $i \neq j$, in the case under discussion include several terms, there are few physically interesting cases in which the expressions simplify and the dependence on the Majorana CP-violation phase(s) and/or on the leptogenesis CPV parameters is prominent.

4.1 Normal Hierarchical Neutrino Mass Spectrum

If the neutrino mass spectrum is of the *normal hierarchical* (NH) type and m_1 is negligibly small, i.e., $|a|\sqrt{m_1 m_2}, |b|\sqrt{m_1 m_3} \ll |c|\sqrt{m_2 m_3}$, the quantities $(Y_\nu^\dagger Y_\nu)_{ij}$, $i \neq j$, depend, in particular, on the same Majorana phase difference $(\alpha - \beta_M)$ on which the effective Majorana mass, eq. (8), depends, on the Dirac CPV phase δ and one leptogenesis CPV parameter, c. The terms $\propto c\sqrt{m_2 m_3} s_{13} e^{-i\delta}$ give always subdominant contributions in $|(Y_\nu^\dagger Y_\nu)_{12,13}|$. For $s_{13} \ll \tan\theta_{12} \sim 0.65$ they are negligible. In this case the expressions for $|(Y_\nu^\dagger Y_\nu)_{12,13}|$ simplify:

$$|(Y_\nu^\dagger Y_\nu)_{12}^{\rm NH}|^2 \cong \frac{M_R^2}{v_u^4} \left| c_{23}\ P^{\rm NH} + s_{23}\ Q^{\rm NH} \right|^2 , \tag{26}$$

$$|(Y_\nu^\dagger Y_\nu)_{13}^{\rm NH}|^2 \cong \frac{M_R^2}{v_u^4} \left| -s_{23}\ P^{\rm NH} + c_{23}\ Q^{\rm NH} \right|^2 , \tag{27}$$

where

$$P^{\rm NH} = (\Delta m_{21}^2)^{\frac{1}{2}} c_{12} s_{12} , \tag{28}$$

$$Q^{\rm NH} = (\Delta m_{31}^2)^{\frac{1}{2}} s_{13} e^{-i\delta} + i\ 2c\ (\Delta m_{21}^2 \Delta m_{31}^2)^{\frac{1}{4}}\ s_{12}\ e^{i\frac{\alpha-\beta_M}{2}} . \tag{29}$$

The double ratio R(21/31) is determined completely by the solar and atmospheric neutrino oscillation parameters Δm_{21}^2, θ_{12} and Δm_{31}^2 and θ_{23}, by the CHOOZ angle θ_{13} and by the Majorana and Dirac CPV phases $(\alpha - \beta_M)$ and δ and by the leptogenesis CPV parameter c:

$$\mathrm{R}(21/31) \cong \frac{\left| c_{23}\ P^{\rm NH} + s_{23}\ Q^{\rm NH} \right|^2}{\left| -s_{23}\ P^{\rm NH} + c_{23}\ Q^{\rm NH} \right|^2} . \tag{30}$$

where $P^{\rm NH}$ and $Q^{\rm NH}$ are given by eqs. (28) and (29). It follows from eqs. (28), (29) and (30) that if $(\alpha - \beta_M) = 0$ and $\delta = \pm\pi/2, \pm 3\pi/2$, we would have

$$\mathrm{R}(21/31) \cong \frac{c_{23}^2 \left| P^{\rm NH} \right|^2 + s_{23}^2 \left| Q^{\rm NH} \right|^2}{s_{23}^2 \left| P^{\rm NH} \right|^2 + c_{23}^2 \left| Q^{\rm NH} \right|^2} . \tag{31}$$

For the best fit value $\sin^2 2\theta_{23} = 1$ we get R(21/31) = 1 independently of the value of the leptogenesis CPV parameter c, although the corresponding branching ratios $\mathrm{BR}(\mu \to e + \gamma)$ and $\mathrm{BR}(\tau \to e + \gamma)$ can exhibit strong dependence on c. If θ_{23} differs somewhat from $\pi/4$, the dependence of R(21/31) on c will, in general, be relatively mild. For $|P^{\rm NH}|^2 \gg |Q^{\rm NH}|^2$

($|P^{\rm NH}|^2 \ll |Q^{\rm NH}|^2$), however, the dependence of R(21/31) on c will be negligible even if $\theta_{23} \neq \pi/4$ and we would have $\mathrm{R}(21/31) \cong \cot^2\theta_{23}$ ($\tan^2\theta_{23}$).

For $|c| \lesssim 0.1$ and s_{13} having a value close to the existing (3σ) upper limit $s_{13} \cong 0.2$, the term $\propto c$ in $Q^{\rm NH}$, eq. (29), gives practically negligible contributions in $|(Y_\nu^\dagger Y_\nu)_{12,13}|$, while $P^{\rm NH}$, eq. (28), gives a subleading but non-negligible contribution. Correspondingly, the double ratio R(21/31) exhibits in this case a significant dependence on the Dirac phase δ and is essentially independent of the leptogenesis CPV parameter c and the Majorana phase $(\alpha - \beta_M)$ (Fig. 1).

If $s_{13} \lesssim 0.1$, but s_{13} is not much smaller than $\sqrt{\Delta m^2_{21}}c_{12}s_{12}/\sqrt{\Delta m^2_{31}} \sim 0.07$, the branching ratios $\mathrm{BR}(\mu \rightarrow e + \gamma)$ and $\mathrm{BR}(\tau \rightarrow e + \gamma)$ still depend on the CHOOZ mixing angle θ_{13} and the phase δ. For $0.01 \lesssim |c| \lesssim 0.10$, $\mathrm{BR}(\mu \rightarrow e + \gamma)$ and $\mathrm{BR}(\tau \rightarrow e + \gamma)$ can exhibit significant dependence also on the Majorana phase $(\alpha - \beta_M)$. The dependence of the double ratio R(21/31) on $(\alpha - \beta_M)$ and δ can be very strong due to possible mutual compensation between $P^{\rm NH}$ and $Q^{\rm NH}$ (see eq. (30)). For $(\alpha - \beta_M) \cong 0$, $s_{13} = 0.10$ and sufficiently small $|c|$, for instance, we can have $\mathrm{R}(21/31) \sim 10^{-2}$ or $\mathrm{R}(21/31) \sim 10^{2}$ depending on whether $\delta \cong \pi$ or $\delta \cong 0$; for $(\alpha - \beta_M) \cong \pi$ and $|c| \sim 0.03$, R(21/31) can have a value $\mathrm{R}(21/31) \sim 10^{-3}$ or $\mathrm{R}(21/31) \sim 10^{3}$, respectively (Fig. 1).

For rather small values of s_{13}, namely, $s_{13} \ll \sqrt{\Delta m^2_{21}}c_{12}s_{12}/\sqrt{\Delta m^2_{31}}$, the dependence of $|(Y_\nu^\dagger Y_\nu)_{12,13}|$ on $s_{13}e^{-i\delta}$ is insignificant and can be neglected. Under the latter condition we also have $\sqrt{\Delta m^2_{31}}s^2_{13} \ll \sqrt{\Delta m^2_{21}}s^2_{12}$. The effective Majorana mass in $(\beta\beta)_{0\nu}$-decay is given correspondingly by $|{<}m{>}|_{\rm NH} \cong \sqrt{\Delta m^2_{21}}s^2_{12}$. The quantities $P^{\rm NH}$ and $Q^{\rm NH}$ can be written as:

$$P^{\rm NH} \cong (|{<}m{>}|_{\rm NH})^{\frac{1}{2}}(\Delta m^2_{21})^{\frac{1}{4}}c_{12}\ ,\ \ Q^{\rm NH} \cong i\ 2c\ (|{<}m{>}|_{\rm NH})^{\frac{1}{2}}(\Delta m^2_{31})^{\frac{1}{4}}\ e^{i\frac{\alpha-\beta_M}{2}}\ . \tag{32}$$

Thus, in this case $\mathrm{BR}(\mu \rightarrow e + \gamma) \propto |{<}m{>}|_{\rm NH}$ and $\mathrm{BR}(\tau \rightarrow e + \gamma) \propto |{<}m{>}|_{\rm NH}$. Given Δm^2_{21}, Δm^2_{31}, θ_{12} and θ_{23}, the ratio R(21/31) is determined by the Majorana phase difference $(\alpha - \beta_M)$ and the leptogenesis CPV parameter c:

$$\mathrm{R}(21/31) \cong \frac{\left|(\Delta m^2_{21})^{\frac{1}{4}}c_{12}\cot\theta_{23} + i\ 2c\ (\Delta m^2_{31})^{\frac{1}{4}}\ e^{i\frac{\alpha-\beta_M}{2}}\right|^2}{\left|(\Delta m^2_{21})^{\frac{1}{4}}c_{12} - i\ 2c\ (\Delta m^2_{31})^{\frac{1}{4}}\ e^{i\frac{\alpha-\beta_M}{2}}\cot\theta_{23}\ \right|^2}\ . \tag{33}$$

Obviously, for $(\alpha - \beta_M) \cong 0$, we have $\mathrm{R}(21/31) \cong 1$. If, however, $(\alpha - \beta_M) \cong \pm\pi$, the double ratio R(21/31) can depend strongly on the value of $|c|$, provided $|c| \gtrsim 0.05$. For $2|c| \sim (\Delta m^2_{21}/\Delta m^2_{31})^{\frac{1}{4}} \cong 0.42$, the two terms in the numerator (denominator) of the expression for R(21/31) can compensate (partially) each other and one can have $\mathrm{R}(21/31) \sim (10^{-3} - 10^{-2})$ or $\mathrm{R}(21/31) \sim (10^{3} - 10^{2})$ depending on the sign of c (Fig. 1).

If $|c|$ is relatively small, $2|c| \ll (\Delta m^2_{21}/\Delta m^2_{31})^{\frac{1}{4}} \cong 0.42$, $\mathrm{BR}(\mu \rightarrow e+\gamma)$ and $\mathrm{BR}(\tau \rightarrow e+\gamma)$ are practically independent of c and $(\alpha - \beta_M)$. This case was analised recently in [30]. If, for instance, $s_{13} \ll \sqrt{\Delta m^2_{21}}c_{12}s_{12}/\sqrt{\Delta m^2_{31}}$, we find from eq. (33):

$$\mathrm{R}(21/31) \cong \cot^2\theta_{23}\ . \tag{34}$$

The results for the double ratio R(21/31) discussed above are illustrated in Fig. 1, where the dependence of R(21/31) on the leptogenesis CPV parameter c for $s_{13} = 0;\ 0.10;\ 0.20$ and few characteristic values of the Majorana and Dirac CPV phases $(\alpha - \beta_M) = 0; \pi/2; \pm\pi$ and $\delta = 0; \pm\pi/2; \pm\pi$ are shown. The figure was obtained using the best fit values of the solar and atmospheric neutrino oscillation parameters θ_{12}, Δm^2_{21}, θ_{23} and Δm^2_{31}. The lightest neutrino mass m_1 was set to 0. The quantities $|(Y^\dagger_\nu Y_\nu)^{\rm NH}_{12,13}|^2$ were calculated using eqs. (15) and (17) and not the approximate expressions given in eqs. (21) and (22). The leptogenesis CPV violating parameters a and b can contribute only to the higher order corrections $\mathcal{O}(r^2, s^2_{13})$ in $|(Y^\dagger_\nu Y_\nu)^{\rm NH}_{12,13}|^2$. These corrections can be relevant for the evaluation of R(21/31) in the case of cancellations between the terms in $|(Y^\dagger_\nu Y_\nu)^{\rm NH}_{12,13}|^2$, which provide the leading order contributions. We have allowed a and b to vary in the same interval as the parameter c in the calculations. The effects of the higher order corrections due to a and b is reflected in the widths of the lines in Fig. 1.

We shall perform next similar analysis for the double ratio R(21/32) = BR($\mu \rightarrow e + \gamma$)/BR($\tau \rightarrow \mu + \gamma$). As can be easily verified using eqs. (21) and (23) and the known values of the neutrino oscillation parameters, for $|c| \leq 0.3$ we always have

$$\mathrm{R}(21/32) < 1 \,. \tag{35}$$

Typically the stronger inequality $\mathrm{R}(21/32) \ll 1$ holds [8] (see further).

It is not difficult to convince oneself also that the term $\propto \Delta_{31} s_{23} c_{23}$ dominates in $|(Y^\dagger_\nu Y_\nu)_{23}|^2$. Indeed, we have $(\Delta m^2_{21}/\Delta m^2_{31})^{\frac{1}{2}} \cong 0.18$, $c_{12}|\cos\theta_{23}| \lesssim 0.24$, $s_{13} s_{12} \sin 2\theta_{23} \lesssim 0.12$, and for $|c| \leq 0.2$ (0.3), the terms $\propto c$ in eq. (23) give contributions which do not exceed approximately 8% (18%). Keeping only the largest of these contributions we have: $|(Y^\dagger_\nu Y_\nu)_{23}|^2 \cong M^2_R v_u^{-4} \, |\sqrt{\Delta m^2_{31}} s_{23} c_{23} + 2ic \, (\Delta m^2_{21} \Delta m^2_{31})^{\frac{1}{4}} c_{12} \cos(\alpha - \beta_M)/2|^2$. Thus, the branching ratio BR($\tau \rightarrow \mu + \gamma$) exhibits very weak dependence on c and $(\alpha - \beta_M)$. Up to the indicated corrections which for $|c| \leq 0.3$ can increase $|(Y^\dagger_\nu Y_\nu)_{23}|^2$ by not more than 18%, we have:

$$|(Y^\dagger_\nu Y_\nu)_{23}|^2 \cong \frac{M^2_R}{v^4_u} \, \Delta m^2_{31} s^2_{23} c^2_{23} \,. \tag{36}$$

Thus, for $|c| \lesssim 0.3$ in the case under discussion, BR($\tau \rightarrow \mu + \gamma$) depends essentially only on the atmospheric neutrino oscillation parameters Δm^2_{31} and θ_{23} (and not on the Dirac and Majorana CPV phases, leptogenesis CPV parameters or solar neutrino oscillation parameters Δm^2_{21} and θ_{12}) and has a relatively simple form: $\mathrm{BR}(\tau \rightarrow \mu + \gamma) \cong \mathrm{F} \times (\Delta m^2_{31}/(4 v^2_u)) \sin^2 2\theta_{23}$, where the factor $\mathrm{F} \propto M^2_R/v^2_u$ contains all the dependence on M_R, $\tan\beta$ and the SUSY breaking parameters (see eq. (11)). The double ratio R(21/32), however, depends in the case under discussion both on $c \, e^{i \frac{\alpha - \beta_M}{2}}$ and $s_{13} e^{-i\delta}$:

$$\mathrm{R}(21/32) \cong \frac{\left| c_{23} \, P^{\rm NH} + s_{23} \, Q^{\rm NH} \right|^2}{\Delta m^2_{31} s^2_{23} c^2_{23}} \,, \tag{37}$$

where $P^{\rm NH}$ and $Q^{\rm NH}$ are given by eqs. (28) and (29).

For $s_{13} \cong 0.2$ and $|c| \lesssim 0.25$, we have $\sqrt{\Delta m^2_{31}} s_{13} \cong 2.3 \sqrt{\Delta m^2_{21}} c_{12} s_{12}$ and $\sqrt{\Delta m^2_{31}} s_{13} \gtrsim 1.7 \, (2c (\Delta m^2_{31} \Delta m^2_{21})^{\frac{1}{4}} s_{12})$. The double ratio R(21/32) exhibits noticeable dependence on the

[8] This is in contrast to the case of normal hierarchical heavy Majorana neutrino mass spectrum, in which one typically has $\mathrm{R}(21/32) \sim 1$ [30].

CPV phases $(\alpha-\beta_M)$ and δ. For $(\alpha-\beta_M)=0$ and $\delta=0$, the term $\propto c$ in $Q^{\rm NH}$ gives a subdominant contribution and R(21/32) is practically independent of c. If $\delta=\pi$, however, the term $\propto \sqrt{\Delta m^2_{31}}s_{13}$ in $Q^{\rm NH}$ can be compensated partially by $P^{\rm NH}$ and for sufficiently large values of $|c|$ the term $\propto c$ in $Q^{\rm NH}$ can be non-negligible. For $|c|\lesssim 0.1$ in this case we can have R(21/32) $\sim$ few $\times 10^{-2}$, while if $(\alpha-\beta_M)=\pi$ and $|c|\cong 0.2$, R(21/32) can be as small as R(21/32) $\sim$ few $\times 10^{-3}$.

In the case of $s_{13}\sim\sqrt{\Delta m^2_{21}}c_{12}s_{12}/\sqrt{\Delta m^2_{31}}\sim 0.07$, partial compensation between the three terms in the numerator of the double ratio R(21/32) can take place. The double ratio R(21/32) can be particularly strongly suppressed for $\delta\cong\pi$, when values of R(21/32) $\sim (10^{-3}-10^{-4})$ for $|c|\sim 0.05$ are possible. Similar mutual compensations between the terms in the numerator of R(21/32) can be realised if $s_{13}\ll\sqrt{\Delta m^2_{21}}c_{12}s_{12}/\sqrt{\Delta m^2_{31}}$ and $|c|\sim (0.15-0.20)$. One can have R(21/32) $\sim (10^{-3}-10^{-4})$ in this case as well. For sufficiently small s_{13} the dependence on the phase δ is obviously insignificant and we have:

$$
{\rm R}(21/32)\cong\frac{|{<\!m\!>}|_{\rm NH}}{\sqrt{\Delta m^2_{31}}s^2_{23}c^2_{23}}\left|\left(\frac{\Delta m^2_{21}}{\Delta m^2_{31}}\right)^{\frac{1}{4}}c_{23}c_{12}+i\;2c\;s_{23}e^{i\frac{\alpha-\beta_M}{2}}\right|^2 . \tag{38}
$$

If in addition $2|c|\ll(\Delta m^2_{21}/\Delta m^2_{31})^{\frac{1}{4}}\cong 0.42$, R(21/32) is also practically independent of c and $(\alpha-\beta_M)$. It is determined completely by the solar and atmospheric neutrino oscillation parameters:

$$
{\rm R}(21/32)\cong|{<\!m\!>}|_{\rm NH}\,\frac{(\Delta m^2_{21})^{\frac{1}{2}}}{\Delta m^2_{31}s^2_{23}}\,c^2_{12}\cong\frac{\Delta m^2_{21}}{\Delta m^2_{31}s^2_{23}}\,c^2_{12}s^2_{12}\simeq 1.3\times 10^{-2}\,. \tag{39}
$$

The specific features of the double ratio R(21/32) discussed above are evident in Fig. 2, where the dependence of R(21/32) on the leptogenesis CPV parameter c, $|c|\leq 0.25$, for three values of $s_{13}=0;\ 0.1;\ 0.2$, and several characteristic values of the Majorana and Dirac CPV phases $(\alpha-\beta_M)$ and δ is shown. Figure 2 was obtained using the same method and the same best fit values of the oscillation parameters Δm^2_{21}, $\sin^2\theta_{12}$, Δm^2_{31} and $\sin^2 2\theta_{23}$, as Fig. 1.

4.2 Inverted Hierarchical Spectrum

If neutrino mass spectrum is *inverted hierarchical* (IH) one has $m_3\ll m_{1,2}$, and we shall assume that the terms $\propto\sqrt{m_3}$ in eqs. (21)-(23) can be neglected. For ${<\!m\!>}_{\rm IH}$ we have ${<\!m\!>}_{\rm IH}\cong\sqrt{|\Delta m^2_{31}|}(c^2_{12}+s^2_{12}e^{i\alpha})$ (see eq. (9)). Now $|(Y^\dagger_\nu Y_\nu)_{ij}|$, $i\neq j$, depend on the Majorana phase α, on the leptogenesis CPV parameter a and, if s_{13} has a value close to the current upper limit - on the Dirac phase δ:

$$
|(Y^\dagger_\nu Y_\nu)^{\rm IH}_{12}|\cong\frac{M_R}{v^2_u}\;\left|c_{23}\;P^{\rm IH}+s_{23}\;Q^{\rm IH}\right|\;, \tag{40}
$$

$$
|(Y^\dagger_\nu Y_\nu)^{\rm IH}_{13}|\cong\frac{M_R}{v^2_u}\;\left|-s_{23}\;P^{\rm IH}+c_{23}\;Q^{\rm IH}\right|\;, \tag{41}
$$

$$
|(Y^\dagger_\nu Y_\nu)^{\rm IH}_{23}|\cong\frac{M_R}{v^2_u}\sqrt{|\Delta m^2_{31}|}\;c_{23}s_{23}\left|-1+4ac_{12}s_{12}\sin\frac{\alpha}{2}\right|\;, \tag{42}
$$

where

$$P^{\rm IH} = \frac{1}{2}\ \frac{\Delta m^2_{21}}{\sqrt{|\Delta m^2_{31}|}}\ c_{12}s_{12} + i\ 2a <m>_{\rm IH}\ e^{-i\frac{\alpha}{2}}\ , \tag{43}$$

$$Q^{\rm IH} = -\sqrt{|\Delta m^2_{31}|}\ s_{13}\ e^{-i\delta}\left(1 + 4ac_{12}s_{12}\ \sin\frac{\alpha}{2}\right)\ . \tag{44}$$

For s_{13} satisfying

$$\sin\theta_{13}(1 + 2|a|\sin 2\theta_{12}) \ll \min\left(2|a|\cos 2\theta_{12}, \frac{\Delta m^2_{21}}{4|\Delta m^2_{31}|}\sin 2\theta_{12}\right) \tag{45}$$

the dependence of $|(Y^\dagger_\nu Y_\nu)^{\rm IH}_{12,13}|$ on the Dirac phase δ would be insignificant. The terms $\propto Q^{\rm IH}$ in eqs. (40) and (41) are negligible, and the ratio of $\mathrm{BR}(\mu \to e + \gamma)$ and $\mathrm{BR}(\tau \to e + \gamma)$ is given by

$$\mathrm{R}(21/31) \cong \cot^2\theta_{23}\ , \tag{46}$$

independently of the values of the Majorana CPV phase α, leptogenesis CPV parameter a, etc. (Fig. 3). If in addition $|a| \ll (\Delta m^2_{21}/(8|\Delta m^2_{31}|))\sin 2\theta_{12} \cong 3.6\times 10^{-3}$, $\mathrm{BR}(\mu \to e + \gamma)$ and $\mathrm{BR}(\tau \to e + \gamma)$ also will not depend on α and a: $|(Y^\dagger_\nu Y_\nu)^{\rm IH}_{12(13)}|^2 \cong C^2_{12(13)}\ (M^2_R\Delta m^2_{21}/v^4_u)(\Delta m^2_{21}/(16|\Delta m^2_{31}|))\sin^2 2\theta_{12}$, where $C_{12(13)} \equiv c_{23}\ (s_{23})$. In the case of $|a|\cos 2\theta_{12} \gg (\Delta m^2_{21}/(8|\Delta m^2_{31}|))\sin 2\theta_{12} \cong 4\times 10^{-3}$, however, we have:

$$|(Y^\dagger_\nu Y_\nu)^{\rm IH}_{12(13)}| \cong 2|a|\ \ |<m>_{\rm IH}|\ \frac{M_R}{v^2_u}\ C_{12(13)}\ . \tag{47}$$

Thus, both $\mathrm{BR}(\mu \to e + \gamma)$ and $\mathrm{BR}(\tau \to e + \gamma)$ are proportional to $|a|^2\ |<m>_{\rm IH}|^2$.

We get $\mathrm{R}(21/31) \sim 1$ also when $s_{13} \gg (\Delta m^2_{21}/(4|\Delta m^2_{31}|))\sin 2\theta_{12} \cong 8\times 10^{-3}$, provided $\alpha \cong 0$ and $\delta \cong 0;\ \pi$ (Fig. 3). In this case $|<m>_{\rm IH}|^2 \cong |\Delta m^2_{31}|$, $|(Y^\dagger_\nu Y_\nu)^{\rm IH}_{12}| \cong (4a^2c^2_{23} + s^2_{13}s^2_{23})|\Delta m^2_{31}|M^2_R/v^4_u$, $|(Y^\dagger_\nu Y_\nu)^{\rm IH}_{13}| \cong (4a^2s^2_{23} + s^2_{13}c^2_{23})|\Delta m^2_{31}|M^2_R/v^4_u$ and

$$\mathrm{R}(21/31) \cong \frac{4a^2c^2_{23} + s^2_{13}s^2_{23}}{4a^2s^2_{23} + s^2_{13}c^2_{23}}\ . \tag{48}$$

If, however, α is significantly different from zero, say $\alpha \cong \pm\pi/2;\ \pm\pi$, and $|a|$ is sufficiently large, being comparable in magnitude to s_{13}, the terms $\propto P^{\rm IH}$ and $\propto Q^{\rm IH}$ in $|(Y^\dagger_\nu Y_\nu)^{\rm IH}_{12}|$ or $|(Y^\dagger_\nu Y_\nu)^{\rm IH}_{13}|$ can partially compensate each other and we can have $\mathrm{R}(21/31) \sim (10^{-3} - 10^{-2})$ or $\mathrm{R}(21/31) \sim (10^2 - 10^3)$ (Fig. 3). For given $|a|$ and s_{13}, the degree of compensation depends on the values of α and δ and on the $\mathrm{sgn}(a)$. It is maximal in $|(Y^\dagger_\nu Y_\nu)^{\rm IH}_{12(13)}|$, for, e.g., $\alpha = \pi$ and $\delta = 0$, or $\alpha = -\pi$ and $\delta = \pi$, and $a > 0$ ($a < 0$) (Fig. 3).

The ratio of $\mathrm{BR}(\mu \to e + \gamma)$ and $\mathrm{BR}(\tau \to \mu + \gamma)$ depends both on a and α. For $|a| \leq 0.3$ we have $\mathrm{R}(21/32) \lesssim 1$; if $|a| \leq 0.1$, the stronger inequality $\mathrm{R}(21/32) \ll 1$ typically holds. For $|a| \ll (\Delta m^2_{21}/(8|\Delta m^2_{31}|))\sin 2\theta_{12} \cong 4\times 10^{-3}$ and negligibly small s_{13}, for instance, one finds [30] $\mathrm{R}(21/32) \cong 10^{-4}$. If, however, the term $\propto a$ dominates in $|P^{\rm IH}|$, i.e., if $|a|\cos 2\theta_{12} \gg 4\times 10^{-3}$, we get (for $s_{13} \sim 0$) $\mathrm{BR}(\mu \to e + \gamma) \propto |a|^2\ |<m>_{\rm IH}|^2$ and correspondingly,

$$\mathrm{R}(21/32) \cong 4\ |a|^2 s^{-2}_{23}\ r_{\rm IH}\left|-1 + 2a\eta\,(1 - r_{\rm IH})^{\frac{1}{2}}\right|^{-2}\ , \tag{49}$$

where $\eta \equiv \mathrm{sgn}(\sin 2\theta_{12} \sin \frac{\alpha}{2})$ and

$$r_{\mathrm{IH}} \equiv \frac{(|<m>|_{\mathrm{IH}})^2}{|\Delta m^2_{31}|} = 1 - \sin^2 2\theta_{12} \sin^2 \frac{\alpha}{2}\ . \tag{50}$$

Now R(21/32) can be considerably larger: for α varying between 0 and π and $|a|$ having a value, e.g., in the interval (0.04 - 0.10), the ratio of interest satisfies $1.9 \times 10^{-3} \lesssim \mathrm{R}(21/32) \lesssim 8.0 \times 10^{-2}$, the maximal value corresponding to $|a| = 0.1$ and $\alpha = 0$.

The predictions for the double ratios R(21/31) and R(21/32), corresponding to IH light neutrino mass spectrum are illustrated in Figs. 3 and 4, respectively. As in the case of Figs. 1 and 2, the quantities $|(Y_\nu^\dagger Y_\nu)^{\mathrm{IH}}_{ij}|^2$, $i \neq j$, have been calculated using eqs. (15) and (17) rather than the approximate expressions given in eqs. (21) and (22). The lightest neutrino mass m_3 set to 0. The leptogenesis CPV parameters b and c, which can contribute only to the higher order corrections in $|(Y_\nu^\dagger Y_\nu)^{\mathrm{IH}}_{ij}|^2$ of interest, were varied in the same interval as the parameter a in the calculations. The effects of the higher order corrections due to b and c is reflected in the widths of the lines in Figs. 3 and 4.

4.3 Quasi-Degenerate Neutrinos

For QD light neutrino mass spectrum, $m_{1,2,3} \cong m \gtrsim 0.1$ eV, one has $<m>_{\mathrm{QD}} \cong m(c^2_{12} + s^2_{12} e^{i\alpha})$, and $\sqrt{m_i m_j} \cong m$ in eqs. (21) - (23). Barring "accidental" cancellations, we always have $|(Y_\nu^\dagger Y_\nu)_{12}| \sim |(Y_\nu^\dagger Y_\nu)_{13}|$, and correspondingly $\mathrm{BR}(\mu \to e + \gamma) \sim \mathrm{BR}(\tau \to e + \gamma)$, in this case (Fig. 5). The expressions for $|(Y_\nu^\dagger Y_\nu)_{12(13)}|$ of interest simplify if $\max(|a|, |b|, |c|) \gg \max(\Delta m^2_{21}/(4m^2), |\Delta m^2_{31}| s_{13}/(4m^2))$ and $s_{13} \lesssim 0.1$:

$$|(Y_\nu^\dagger Y_\nu)^{\mathrm{QD}}_{12}| \cong 2\ \frac{M_R}{v_u^2}\ \left| c_{23}\ P^{\mathrm{QD}} + s_{23}\ Q^{\mathrm{QD}} \right|\ , \tag{51}$$

$$|(Y_\nu^\dagger Y_\nu)^{\mathrm{QD}}_{13}| \cong 2\ \frac{M_R}{v_u^2}\ \left| -s_{23}\ P^{\mathrm{QD}} + c_{23}\ Q^{\mathrm{QD}} \right|\ , \tag{52}$$

where

$$P^{\mathrm{QD}} = a\ <m>_{\mathrm{QD}}\ e^{-i\frac{\alpha}{2}}\ , \tag{53}$$

$$Q^{\mathrm{QD}} = m \left[\left(bc_{12} + cs_{12} e^{i\frac{\alpha}{2}} \right) e^{-i\frac{\beta_M}{2}} + ias_{13} \sin 2\theta_{12}\ e^{-i\delta} \sin \frac{\alpha}{2} \right]\ . \tag{54}$$

The condition specified above is compatible with the leptogenesis constraints on the product $|abc|$ [26]. For $\alpha \cong 0$ and $\beta_M \cong \pm\pi$ we get:

$$\mathrm{R}(21/31) \cong \frac{c^2_{23} \left| P^{\mathrm{QD}} \right|^2 + s^2_{23} \left| Q^{\mathrm{QD}} \right|^2}{s^2_{23} \left| P^{\mathrm{QD}} \right|^2 + c^2_{23} \left| Q^{\mathrm{QD}} \right|^2}\ . \tag{55}$$

Obviously, in this case either $\mathrm{R}(21/31) \cong 1$ independently of the value of θ_{23}, or $\mathrm{R}(21/31) \cong \tan^2\theta_{23}$ or $\cot^2\theta_{23}$.

Under the condition leading to eqs. (51) - (54), the quantity $|(Y_\nu^\dagger Y_\nu)_{23}|$, eq. (23), cannot be simplified. The term $\propto \Delta_{31}$ in eq. (23) will be the dominant one if $\max(|a \sin(\alpha/2)|, 2|a| s_{13}, |a|^2), |b|, |c| \ll |\Delta m^2_{31}|/(4m^2)$. Given the leptogenesis constraint on

$|abc|$, this is realised, e.g., for $m = 0.1$ eV if $|a| \sim |b| \sim |c| \cong 10^{-2}$, or if $\sin(\alpha/2) \cong 0$, $s_{13} \cong 0$ and $|a| \gg |b|, |c|$ but $|a|^2 \ll |\Delta m^2_{31}|/(4m^2)$. In both cases we have

$$\mathrm{R}(21/32) \cong \frac{16\ m^4\ |a|^2}{(\Delta m^2_{31})^2} \left| \frac{<m>_{\mathrm{QD}}}{m\ s_{23}} + \frac{bc_{12} + cs_{12}e^{i\frac{\alpha}{2}}}{a\ c_{23}} e^{-i\frac{\beta_M - \alpha}{2}} \right|^2 \ll 1\ . \tag{56}$$

For $m = 0.10$ eV and $|a| = |b| = |c| = 10^{-2}$, the ratio R(21/32) given by eq. (56) depends on α, β_M, sgn(b/a) and sgn(c/a) and satisfies $2 \times 10^{-4} \lesssim \mathrm{R}(21/32) \lesssim 3 \times 10^{-1}$. If, however, $\alpha \cong 0$, $s_{13} \cong 0$ and $|a| \cong 0.2$ with $|abc| \cong 10^{-5}$, the "corrections" $\propto |a|^2$ in eq. (23) will be non-negligible since $|a|^2 \sim |\Delta m^2_{31}|/(4m^2)$. In this case we can have even $\mathrm{R}(21/32) \cong 200$ as a consequence of rather strong partial cancellation between the different terms in the expression for $|(Y^\dagger_\nu Y_\nu)_{23}|$ (Fig. 6).

The term $\propto \Delta_{31}$ in eq. (23) can be neglected if, e.g., at least one of the CPV parameters $|a\ \sin(\alpha/2)|$, $|b\ \sin(\beta_M/2)\cos 2\theta_{23}|$ ($|b\ \cos(\beta_M/2)|^2$) and $|c\ \sin((\alpha-\beta_M)/2)\cos 2\theta_{23}|$ ($|c\ \cos((\alpha-\beta_M)/2)|^2$) is much bigger than $|\Delta m^2_{31}|/(4m^2)$ ($|\Delta m^2_{31}|^2/(4m^2)^2$). In this case eqs. (51) - (54) are also valid. We get particularly simple expressions for $|(Y^\dagger_\nu Y_\nu)^{\mathrm{QD}}_{ij}|$, $i \neq j$, if the terms $\propto a$ in eqs. (21) - (23) dominate:

$$|(Y^\dagger_\nu Y_\nu)^{\mathrm{QD}}_{12}| \cong 2\ |a|\ \frac{M_R}{v_u^2}\ |<m>|_{\mathrm{QD}}\ c_{23}\ , \tag{57}$$

$$|(Y^\dagger_\nu Y_\nu)^{\mathrm{QD}}_{13}| \cong |(Y^\dagger_\nu Y_\nu)^{\mathrm{QD}}_{12}|\ \tan\theta_{23}\ , \tag{58}$$

$$|(Y^\dagger_\nu Y_\nu)^{\mathrm{QD}}_{23}| \cong 2\ |a|\ \frac{M_R}{v_u^2}\ \sqrt{m^2 - |<m>|^2_{\mathrm{QD}}}\ c_{23}s_{23}\ . \tag{59}$$

Equations (57) - (58) are valid provided $|a| \gg \max(|b|, |c|, |\Delta m^2_{31}|/(4m^2))$, while eq. (59) holds if $|a\ \sin(\alpha/2)| \gg \max(|b|, |c|, |\Delta m^2_{31}|/(4m^2))$. For $|a| < 1$ and, e.g., $m \cong 0.1$ eV, the latter condition requires $|\sin(\alpha/2)| \cong 1$. In these cases both $\mathrm{BR}(\mu \to e + \gamma) \sim |a|^2 |<m>|^2_{\mathrm{QD}}$ and $\mathrm{BR}(\tau \to e + \gamma) \sim |a|^2 |<m>|^2_{\mathrm{QD}}$, while $\mathrm{BR}(\tau \to \mu + \gamma) \sim |a|^2\ (m^2 - |<m>|^2_{\mathrm{QD}}) \cong |a|^2\ m^2 \sin^2 2\theta_{12} \sin^2(\alpha/2)$. For the ratio of the first two we get

$$\mathrm{R}(21/31) \cong \cot^2\theta_{23}\ , \tag{60}$$

which should be compared with eqs. (34) and (46). The ratio of $\mathrm{BR}(\mu \to e + \gamma)$ and $\mathrm{BR}(\tau \to \mu + \gamma)$ is independent of the leptogenesis CPV parameter a. Given θ_{12} and θ_{23}, it is determined by the Majorana phase α:

$$\mathrm{R}(21/32) \cong \frac{|<m>|^2_{\mathrm{QD}}}{(m^2 - |<m>|^2_{\mathrm{QD}})\ s^2_{23}} \cong \frac{1 - \sin^2 2\theta_{12}\sin^2(\alpha/2)}{s^2_{23}\ \sin^2 2\theta_{12}\sin^2(\alpha/2)}\ . \tag{61}$$

We get similar results if the terms $\propto b$ ($\propto c$) dominate in eqs. (21) - (23), which in the case of eq. (23) would be possible only if the Majorana phase β_M (phase difference $\alpha - \beta_M$) deviates significantly from π. Now $|(Y^\dagger_\nu Y_\nu)^{\mathrm{QD}}_{23}|$ depends on β_M ($\alpha - \beta_M$). If, e.g, the terms $\propto c$ dominate we get: $\mathrm{R}(21/32) \cong s^2_{23}\tan^2\theta_{12}\ (1 - \sin^2 2\theta_{23}\sin^2((\alpha-\beta_M)/2))^{-1}$.

Our results for the double ratios R(21/31) and R(21/32) are illustrated in Fig. 5.

5 Conclusions

Working in the framework of the class of SUSY theories with see-saw mechanism and soft SUSY breaking with flavour-universal boundary conditions at a scale $M_X > M_R$, we have analysed the dependence of the rates of lepton flavour violating (LFV) decays $\mu \to e+\gamma$, $\tau \to e+\gamma$ $\tau \to \mu+\gamma$ ($l_i \to l_j + \gamma$) and of their ratios, on the Majorana and Dirac CP-violation (CPV) phases in the PMNS matrix $U_{\rm PMNS}$, α, β_M and δ, and on the leptogenesis CP-violating (CPV) parameters. The case of quasi-degenerate in mass heavy RH neutrinos was investigated, $M_1 \cong M_2 \cong M_3 \equiv M_R$. Results for the normal hierarchical (NH), inverted hierarchical (IH) and quasi-degenerate (QD) light neutrino mass spectra have been derived. The analysis was performed under the condition of negligible renormalization group (RG) effects for the light neutrino masses m_j and the mixing angles and CPV phases in $U_{\rm PMNS}$ [9]. In the wide region of validity of eqs. (11) and (13) in the relevant SUSY parameter space, the ratios of rates of the decays $\mu \to e+\gamma$ and $\tau \to e+\gamma$, and $\mu \to e+\gamma$ and $\tau \to \mu+\gamma$, are independent of the SUSY parameters - they are determined by the neutrino masses (m_j) and mixing angles, Majorana and Dirac CPV phases and by the leptogenesis CPV parameter(s). For the matrix of neutrino Yukawa couplings, $\mathbf{Y}_\nu$ - a basic quantity in the analysis performed, we have used the orthogonal parametrisation [23]. The latter proved to be the most convenient for the purposes of our study [26]. In this parametrisation $\mathbf{Y}_\nu$ is expressed in terms of the light and heavy Majorana neutrino masses, $U_{\rm PMNS}$, and an orthogonal matrix $\mathbf{R}$ [23]. Leptogenesis can take place only if $\mathbf{R}$ is complex. For quasi-degenerate heavy Majorana neutrinos only the three leptogenesis CPV real dimensionless parameters of the matrix $\mathbf{R}$, a, b and c (eqs. (17) and (19)), enter into the expressions for the $l_i \to l_j + \gamma$ decay branching ratios of interest [26], BR($l_i \to l_j + \gamma$). In our analysis we have assumed that $|a|, |b|, |c| < 1$, as is suggested by the leptogenesis constraint derived for QD light neutrinos [26]. In various estimates we have considered values of $|a|, |b|, |c| \leq 0.3$.

We have found that for NH (IH) spectrum and negligible lightest neutrino mass m_1 (m_3), the branching ratios BR($l_i \to l_j + \gamma$) depend, in general, on one Majorana and the Dirac CPV phases, $\alpha - \beta_M$ (α) and δ, one leptogenesis CPV parameter, c (a), on the CHOOZ angle θ_{13} and on the mixing angles and mass squared differences associated with solar and atmospheric neutrino oscillations, θ_{12}, Δm^2_{21}, and θ_{23}, Δm^2_{31}. The double ratios R(21/31) $\propto$ BR($\mu \to e+\gamma$)/BR($\tau \to e+\gamma$) and R(21/32) $\propto$ BR($\mu \to e+\gamma$)/BR($\tau \to \mu+\gamma$) (see eq. (25)) are determined by these parameters. The same Majorana phase $\alpha - \beta_M$ (α) enters also into the NH (IH) expression for the effective Majorana mass in neutrinoless double beta ($(\beta\beta)_{0\nu}$-) decay, $<m>$ (eqs. (8) and (9)). For the QD spectrum, BR($l_i \to l_j+\gamma$) depend, in general, on the absolute neutrino mass m, the three leptogenesis CPV parameters, a, b, c and on the two Majorana phases α and β_M. For the IH and QD spectra, the phase α enters into the expressions for BR($\mu(\tau) \to e+\gamma$), in particular, through the effective Majorana mass $<m>$ (see eqs. (43) and (53)). Our results for the double ratios show that we can have R(21/31) ~ 1 or R(21/31) $\ll 1$, or else R(21/31) $\gg 1$ in the cases of NH and IH spectra, while for the QD spectrum typically R(21/31) ~ 1. In contrast, for the NH and IH spectra one always gets R(21/32) < 1; in most of the relevant parameter space R(21/32) $\ll 1$

[9] It is well-known that in the class of SUSY theories considered, this condition is satisfied in the cases of NH and IH light neutrino mass spectra; it is fulfilled for the QD spectrum provided the SUSY parameter $\tan\beta$ is relatively small, $\tan\beta < 10$.

holds. For the QD spectrum, however, R(21/32) $\gtrsim 1$ is also possible.

More specifically, we find that for the NH (IH) spectrum, BR($\mu(\tau) \rightarrow e + \gamma$) exhibit significant dependence on the leptogenesis CPV parameter c (a) and on the Majorana CPV phase $\alpha - \beta_M$ (α) for $|c| \gtrsim 0.02$ ($|a| \gtrsim 0.02$) and for any $s_{13} \lesssim 0.1$ ($s_{13} \lesssim 0.2$). In certain cases the dependence of BR($\mu(\tau) \rightarrow e + \gamma$) on the phase $\alpha - \beta_M$ (α) and/or the parameter c (a) is dramatic. More generally, the dependence of BR($\mu(\tau) \rightarrow e + \gamma$) on the Majorana phase can be noticeable only if the corresponding leptogenesis parameter is sufficiently large: for $|c| \ll \max((\Delta m^2_{31}/\Delta m^2_{21})^{\frac{1}{4}} s_{13}, 0.5(\Delta m^2_{21}/\Delta m^2_{31})^{\frac{1}{4}})$ in the NH case, and $|a| \ll \max(s_{13}/(2\cos 2\theta_{12}), \Delta m^2_{21} \tan 2\theta_{12}/(8|\Delta m^2_{31}|)$ in the IH one, both c (a) and the Majorana phase have practically no effect on BR($\mu(\tau) \rightarrow e + \gamma$). Similarly, the CHOOZ angle θ_{13} and the Dirac phase δ can be relevant in the evaluation of BR($\mu(\tau) \rightarrow e + \gamma$) in the cases of NH and IH spectra only if s_{13} is large enough, i.e., if respectively, $s_{13} \gtrsim \sqrt{\Delta m^2_{21}} \sin 2\theta_{12}/(2\sqrt{\Delta m^2_{31}|}) \cong 0.07$, and $s_{13} \gtrsim \Delta m^2_{21} \sin 2\theta_{12}/(2|\Delta m^2_{31}|) \cong 8 \times 10^{-3}$. In the case of NH (IH) spectrum, BR($\tau \rightarrow \mu + \gamma$) is practically independent of $s_{13} \lesssim 0.2$; the dependence of BR($\tau \rightarrow \mu + \gamma$) on the leptogenesis parameter c (a) and the Majorana phase $\alpha - \beta_M$ (α) is relatively weak for $|c| \lesssim 0.3$ ($|a| \lesssim 0.1$). For this wide range of values of $|c|$ ($|a|$) we have BR($\tau \rightarrow \mu + \gamma$) $\cong$ F $\times$ $(|\Delta m^2_{31}|/(4v_u^2))\ \sin^2 2\theta_{23}$, where the factor F $\propto M_R^2/v_u^2$ contains all the dependence on M_R, $\tan\beta$ and the SUSY breaking parameters (see eq. (11)).

The double ratios R(21/31) and R(21/32) (Figs. 1 - 4) can exhibit in the cases of NH and IH spectra strong dependence on the Dirac and/or Majorana phases if $s_{13} \sim 0.1 - 0.2$ and/or if the relevant leptogenesis parameter exceeds approximately 10^{-2}. Under the indicated conditions values of R(21/31) $\sim (10^{-3} - 10^{-2}) \ll 1$ or R(21/31) $\sim (10^3 - 10^2) \gg 1$, are possible. For, e.g., $s_{13} \sim 0.1$, the sign of the inequality is determined by the sign of the leptogenesis parameter, the value of the Majorana phase and/or the value of the Dirac phase (Figs. 1 and 3). If for the NH (IH) spectrum, $\alpha - \beta_M \cong 0$ ($\alpha \cong \pi$) and $\delta \cong \pm\pi/2, \pm 3\pi/2$, R(21/31) takes one of the following three values R(21/31) $\cong 1; \tan^2\theta_{23}; \cot^2\theta_{23}$. For $|a| \gg \Delta m^2_{21} \tan 2\theta_{12}/(8|\Delta m^2_{31}|)$ in the IH case, we find BR($\mu(\tau) \rightarrow e+\gamma$) $\cong F^{\rm IH} \times |a|^2\ |<m>_{\rm IH}|^2/v_u^2$, and thus R(21/31) $\cong 1$, where $<m>_{\rm IH}$ is the effective Majorana mass in $(\beta\beta)_{0\nu}$-decay and the factor $F^{\rm IH} \propto M_R^2/v_u^2$ includes the dependence on M_R and on the SUSY parameters. For sufficiently small s_{13} and $|c|$ ($s_{13} \ll 0.07$, $|c| \ll 0.2$)) in the case of NH spectrum, we get: R(21/32) $\cong \Delta m^2_{21}/(\Delta m^2_{31} s^2_{23})\ c^2_{12} s^2_{12} \cong 10^{-2}$. Smaller values of R(21/32) are possible, e.g., for $s_{13} \cong (0.1 - 0.2)$, if $|c| \sim 0.05$ and if for given sgn(c), the Majorana and Dirac phases ($\alpha - \beta_M$) and δ have specific values (Fig. 2). For the IH spectrum we typically have R(21/32) $\ll 1$ for $|a| \leq 0.1$. If $2|a| \cos 2\theta_{12}, \sin\theta_{13} \ll 0.5 \Delta m^2_{21}\ c_{12} s_{12}/|\Delta m^2_{31}|$, R(21/32) is completely determined by the solar and atmospheric neutrino oscillation parameters Δm^2_{21}, θ_{12}, Δm^2_{31} and θ_{23}, and R(21/32) $\cong 10^{-4}$.

In the case of QD light neutrino mass spectrum, the leptogenesis constraint implies [26] $10^{-6} \lesssim |abc| \lesssim 10^{-4}$. The expressions for BR($l_i \rightarrow l_j + \gamma$) and for the double ratios R(21/31) and R(21/32) simplify considerably if the terms including one given leptogenesis parameter dominate. We get, e.g., R(21/31) $\cong \tan^2\theta_{23}$ and R(21/32) $\cong 1$ if the terms $\propto b$ ($\propto c$) are the dominant one. This requires relatively large values of $|a|$ or $|b|$ or $|c|$. If, however, $|a| \sim |b| \sim |c| \simeq 10^{-2}$, R(21/32) lies in the interval $\sim (10^{-4} - 10^{-1})$.

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Note Added. During the completion of the present study we became aware [52] that an analysis along seemingly similar lines is being performed by R. Rückl et al.

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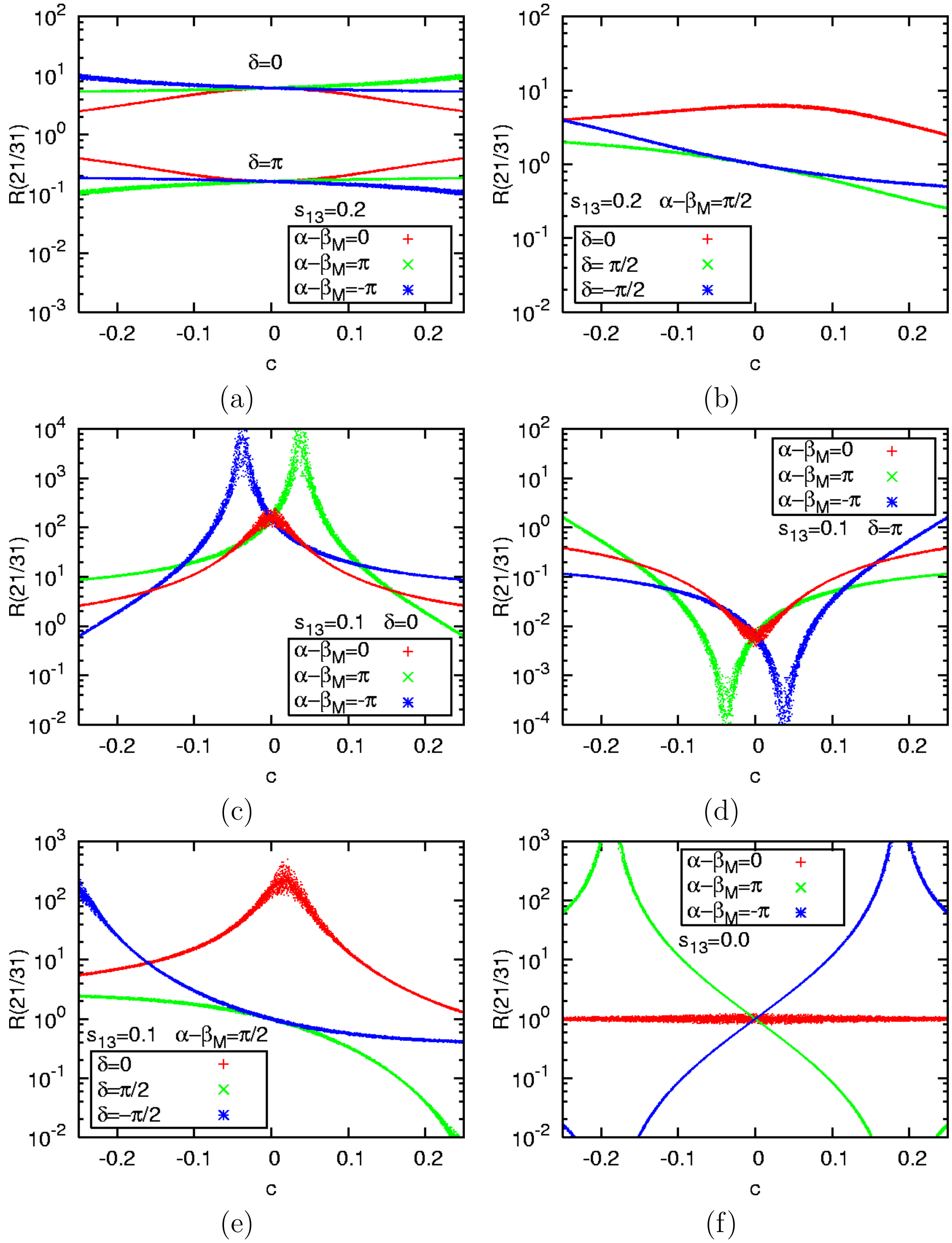


Figure 1: The double ratio R(21/31) in the case of NH light neutrino mass spectrum, as a function of the leptogenesis CPV parameter c, for $s_{13} = 0.2;\ 0.1;\ 0$ and several characteristic values of the Dirac and Majorana CPV phases δ and $\alpha - \beta_M$. The figure was obtained using the best fit values of the solar and atmospheric neutrino oscillation parameters Δm^2_{21}, $\sin^2\theta_{12}$, Δm^2_{31} and $\sin^2 2\theta_{23}$. The lightest neutrino mass m_1 was set to 0. The effects of the higher order corrections in leptogenesis CPV parameters is reflected in the width of the lines (see text for further details).

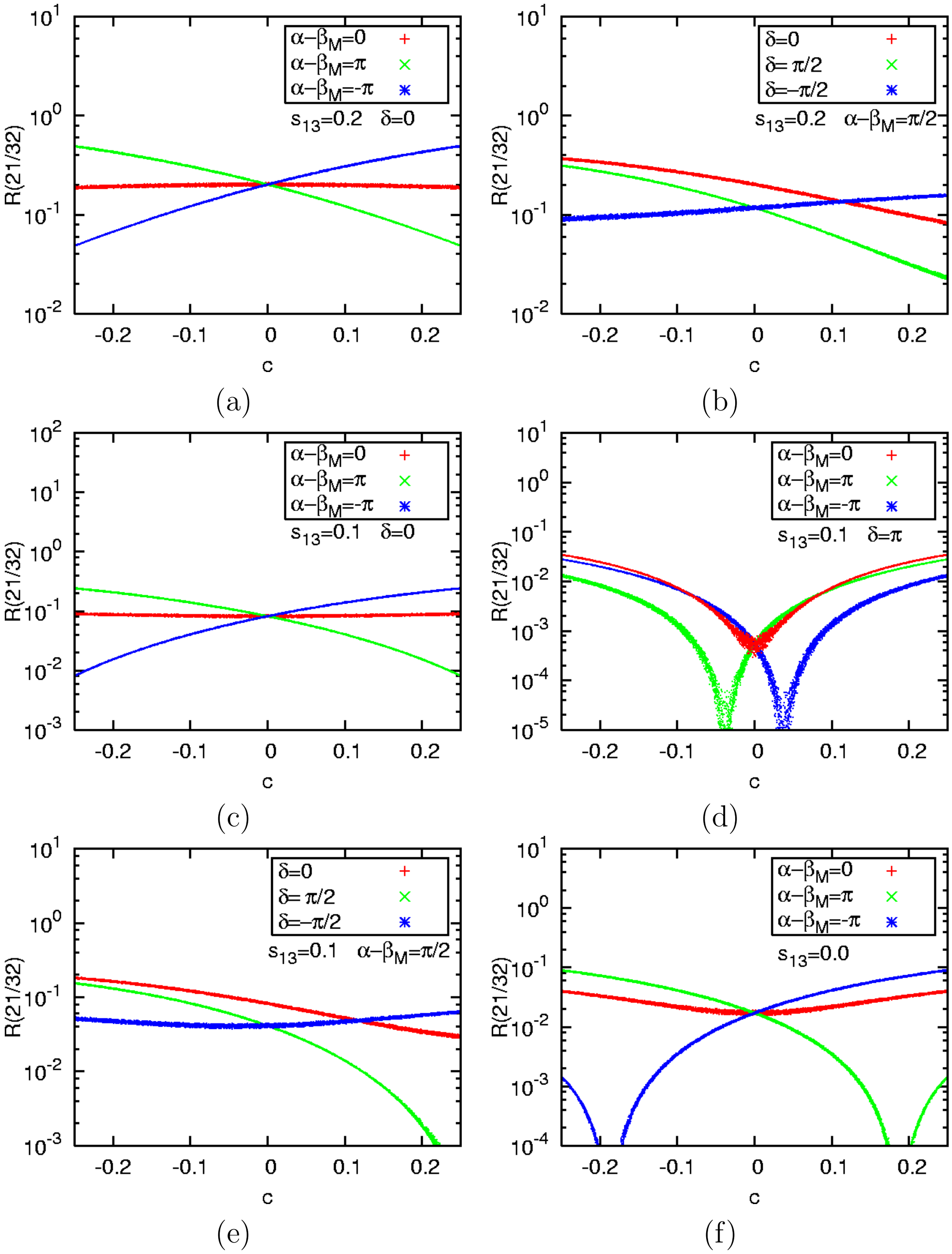


Figure 2: The same as in Fig. 1, but for the double ratio R(21/32).

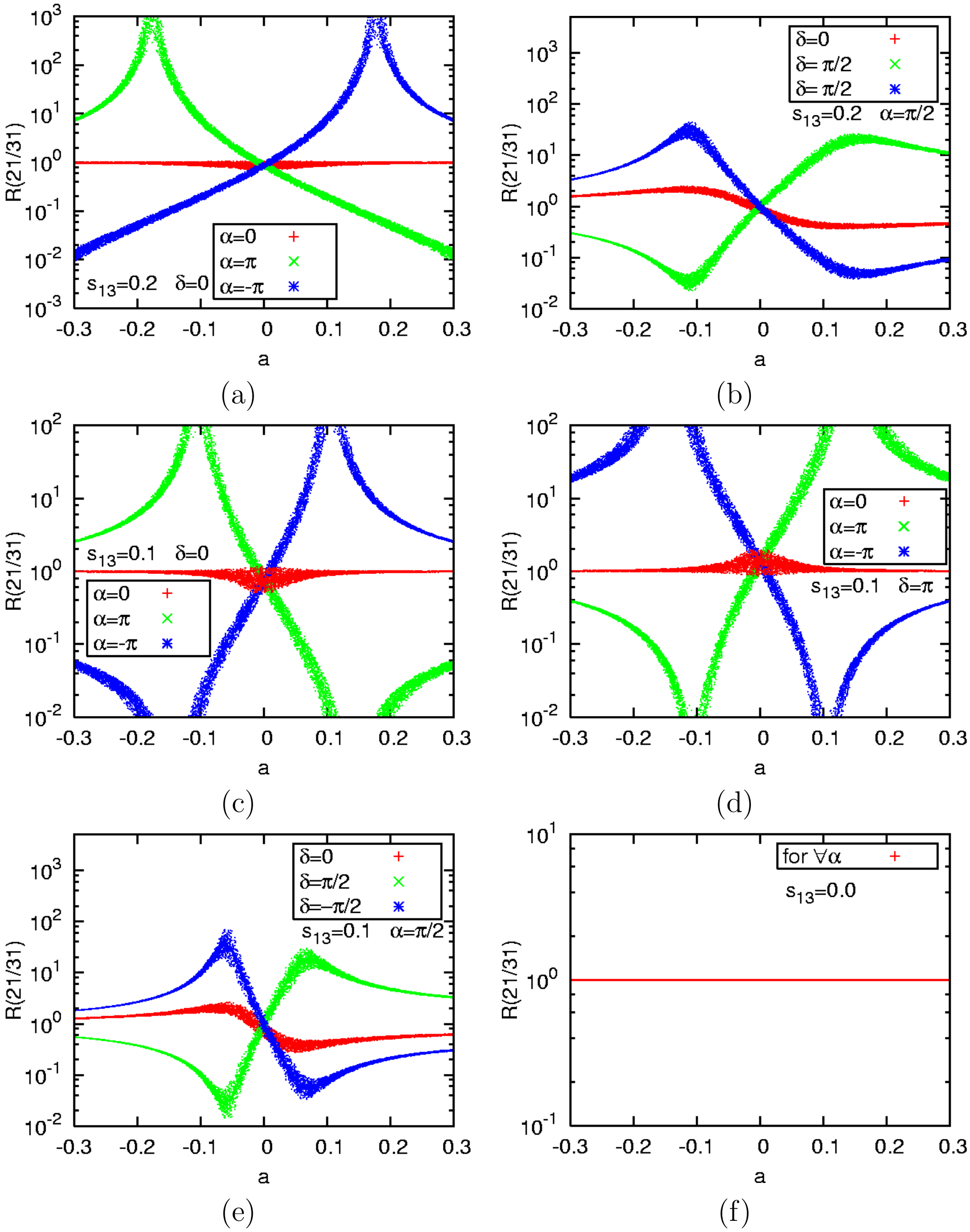


Figure 3: The double ratio R(21/31) in the case of IH light neutrino mass spectrum as a function of the leptogenesis CPV parameter a, for several characteristic values of the CHOOZ angle θ_{13} and Majorana and Dirac CPV phases α and δ. The results shown correspond to the lightest neutrino mass $m_3 = 0$. The effects of the higher order corrections due to the leptogenesis CPV parameters b and c is reflected in the widths of the lines (see text for further details).

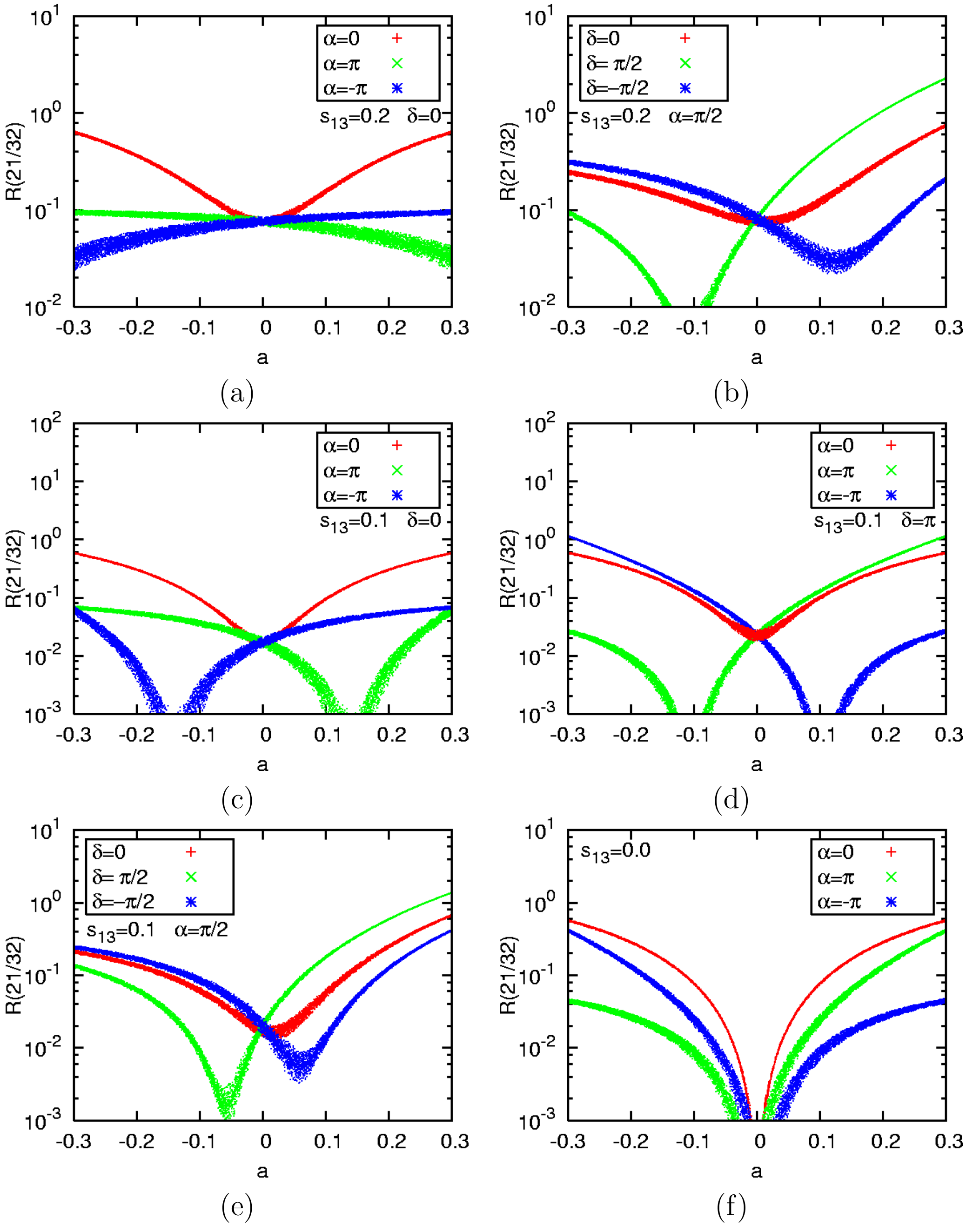
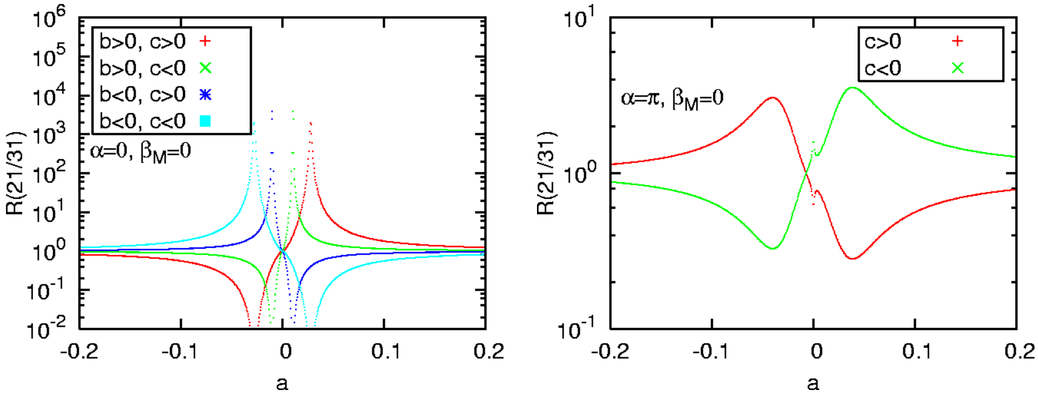
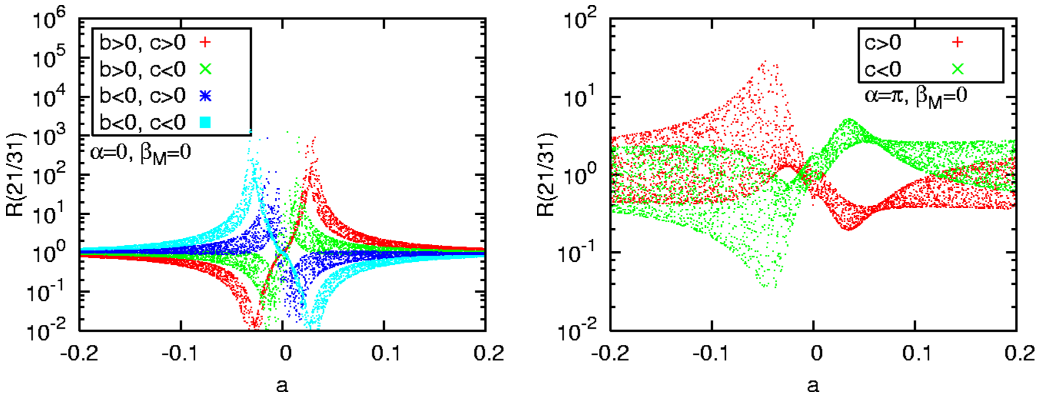


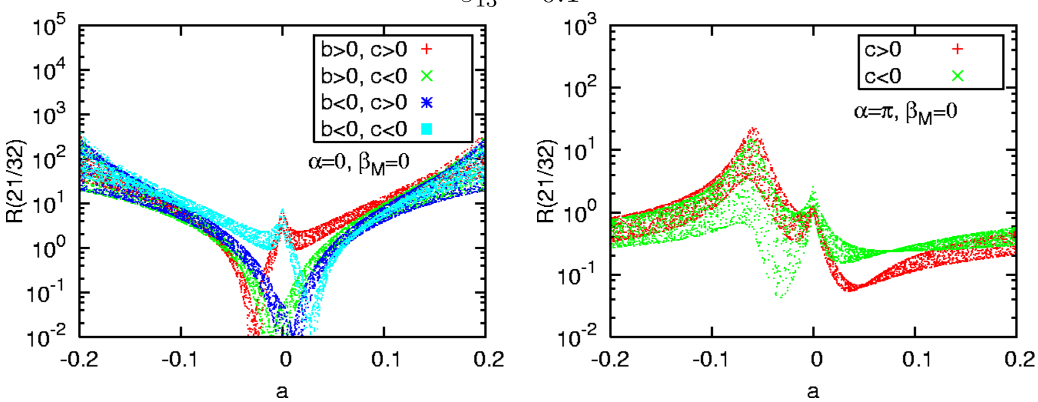
Figure 4: The same as in Fig. 3, but for the double ratio R(21/32).



$s_{13} = 0.0$



$s_{13} = 0.1$



$s_{13} = 0.1$

Figure 5: The double ratios R(21/31) and R(21/32) as a function of the leptogenesis CPV parameter a in the case of QD light neutrino mass spectrum for $s_{13} = 0; 0.1$ and several values of the Majorana phases α and β_M. The results shown are obtained for $|abc| = 10^{-5}$ and $|b| = |c|$. For $s_{13} = 0.1$, values of the Dirac CPV phase $0 \leq \delta \leq \pi$ were considered. The lightest neutrino mass is set to $m_1 = 0.1$ eV.